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Rapport de stage

Transmission Line Cost Allocation, Efficiency and Strategic Behaviour

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Allocation des coûts des lignes de transmission, Efficacité et Comportements Stratégiques

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NON CONFIDENTIEL

Option :	Mathématiques Appliqués
Champ de l′option :	Optimisation et théorie des jeux
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Preface

This thesis is the report of a 3-month research project required for the engineering degree of the Ecole Polytechnique of France. The work was conducted in the University of Auckland during the period from April 2010 to July 2010 in the Engineering Science Department, in the field of mathematics applied to electricity optimization.

The project deals with the analysis of a market design of the New Zealand electricity market. The use of game theory and other mathematical tools enables a sophisticated criticism. The purpose of my research is to show how the market design can lead to strategic behaviours, which can produce instability, inefficiency and market power.

Because the design is recent and confidential, there is not a lot of litterature concerning this specific project. However, game problems in electricity markets are burning issues, as most countries have a very recently liberalized electricity market. In particular, the European electricity market is a hot topic right now. Several mathematical tools have been created to study these markets, and some of them have been used in this paper too.

During this internship, I had quite a lot of liberty. I had the opportunity to take plenty of initiatives. However, I also had the support of my supervisor, Dr. Andy Philpott, who has oriented me all along, and given me the mathematical tools that was required for the completion of my research project.

I wish to express my sincere thanks to Dr. Andy Philpott, for his ever encouraging support, his remarks during our discussions and his advices in the redaction of this report. With his help, not only have I learnt a lot about electricity market and game theory during this period, but I have also had the chance to discover the inside of a research unit in a very international environment.

I would also like to thank him and Dr. Frédéric Bonnans, my supervisor at the Ecole Polytechnique, for giving me the opportunity to complete this internship in the amazing country that New Zealand is. In addition, I acknowledge the support of the chair Optimization for Sustainable Development between Microsoft France, Microsoft Research, the Ecole Polytechnique and the ST2I institute of CNRS.

In addition, I am grateful to Dr. Golbert Zakeri and Jonas Villumsen for helping me in my work but also for welcoming me in the working environment. Finally, I thank the other students that have given me their attention and their support.

Before the end of my internship, a publication of my results is planned, as well as a seminar.

Abstract

The inter-island HVDC line is a major transmission line in New Zealand, as it is the only link between its two islands. It enables the exportation of electricity from the South Island to the North Island. These exportations highly benefit both the generators of the South Island and the consumers of the North Island. Right now, South Island generators are charged for the entire cost of the HVDC line according to a very controversial allocation system. The extra cost incurred by South Island generators is not only claimed by some to be inequitable, but it provides a possibly perverse incentive to build new generation in the North Island, which may be more expensive.

A modification of the current allocation process has been proposed. Generators would be required to bid both for electricity supply and for HVDC line rights. In this paper, we study, given the allocation process, what kind of strategies generators may be tempted to follow, with the use of game theory and the study of supply function equilibria. We show how unstability, inefficiency and market power can result from this process.

Index Terms - Transmission constraints, linear optimization, cost allocation, game theory, Nash Equilibrium, supply function equilibrium.

Résumé

La ligne HVDC inter-island est une ligne de transmission électrique majeure en Nouvelle-Zélande, car elle est la seule à rejoindre les deux îles qui composent le pays. Elle permet ainsi l'exportation de l'électricité de l'île du sud vers l'île du nord, ce qui profite grandement aux générateurs du sud et aux consommateurs du nord. Au jour d'aujourd'hui, les générateurs du sud paient leur utilisation de la ligne HVDC selon un système d'affectation des prix très controversé. Non seulement les générateurs du sud se plaignent de l'inéquité de cette allocation, mais en plus elle incite les générateurs à s'implanter sur l'île du nord plutôt que sur l'île du sud, même si leurs coûts de production y sont plus importants.

Un système d'affectation alternatif a été proposé. Dans ce système, les générateurs se doivent de déclarer à la fois leurs offres en électricité et leurs demandes en droits d'utilisation de la ligne HVDC. Dans ce rapport, nous étudierons, étant donné ce système, les strategies que les générateurs seront amenés à suivre, à l'aide de la théorie des jeux et de l'étude des équilibres de fonctions d'offre. Nous démontrerons ainsi l'instabilité, l'inefficacité et les pouvoirs de marché conséquents qui en resultent.

Mots clés - Contraintes électriques, optimisation linéaire, affectation des coûts, théorie des jeux, équilibre de Nash, équilibre de fonctions d'offres.

Contents

Chapter 1

Introduction

The efficiency of electricity generation and transmission has become an important topic as regions around the world seek to reduce their energy costs. Until recently, electricity generation, transmission, distribution and retailing was run by a central (often government-owned) body. However, many countries have undertaken a restructuring of their electricity systems by removing the vertical integration of generation, transmission, distribution and retailing and attempting to replace it with separate horizontal layers, each made up of several competing organizations. Such competition has been introduced in the expectation that it will lead to an increase of efficiency and a decrease in electricity prices. The hope is also to provide significant signals for investments and new entry.

However, because of its physical properties, electricity transmission is considered to be a natural monopoly. Thus, many countries like England, Wales, Australia and some parts of America (as opposed to California) have decided to structure their market as electricity pools, where transmission is managed by a single entity often called the *Independent System Operator* (or *ISO*) . Given demands of retail companies and wholesale purchasers, the ISO takes into account offers from generators and uses an optimization program that chooses the dispatches that minimize the total revealed cost of power delivery. The program also gives the prices of electricity for both generators and purchasers that may depend on their location.

In New Zealand, the transmission network is owned and operated by a state-owned company called Transpower, whereas the generators are generally owned by a small group of private companies, and state-owned enterprises. The generators inject power into the network, which will supply the nodes of the network. At each node, some of this energy is consumed either by large purchasers or by retail companies that on-sell it to households and small industries. Any net surplus of energy at a node is transmitted through the network to nodes with an energy deficit. Since the transmission of power incurs energy losses, the price of energy is generally different from node to node.

A major component of the New Zealand transmission system is the High-Voltage Direct Current (HVDC) line joining the South Island to the North Island (see Figure 1). This was commissioned in 1965 (and upgraded in 1993) with the main purpose of conveying hydro-electric power from the South Island to the population centres in the North Island. Since the South Island generators tend to export electricity to the North Island, they decrease the North Island prices of electricity, but these generally remain higher than in the South. The benefits for South Island generators based on the higher prices they can earn in the North due to the HVDC line is estimated to be around NZ$240 million per year.

In its standard configuration the HVDC line consists of two parallel cables (or *poles*) although one of these is currently being repaired. The costs of maintaining and improving the HVDC line are borne by Transpower, who recover them by charging the users. These costs are estimated to be approximately NZ$88 million per year. Currently the charge for use is allocated entirely to South Island generators, in proportion to their last 12 month's peak generation. Although it has been in use for some years, this charging scheme has understandably been unpoular with South Island generators, who view it as inequitable. Their claims are given some validity by observing that the HVDC link carries flows from North to South in winters when the hydro-electric lake levels are low (which has occurred in four of the last ten years).

Transmission cost allocation is a complicated problem, as it leads to games. Problems of fixed-cost allocation have a long history in economics, going back to the seminal work of Nash [6], Shapley [10], and Aumann [9] in cooperative game theory. However these methods have not received much attention in the electricity market literature perhaps because of their complexity. By considering a coalition formation, [12] shows that an allocation using the Shapley value has the advantage of being simple, transparent, fair and gives adequate price signals, although it may lead to a difficult mathematical resolution. In [11], the authors show how, in a case where three plants have peaks at different time, a cooperative game can deeply reduce their cost, if the allocation is not fair. In [4], it is explained how restructured energy

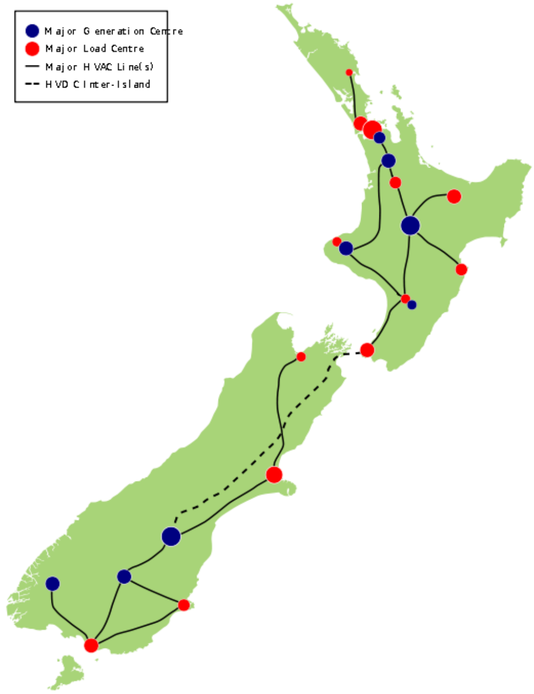

Figure 1.1: New Zealand transmission grid

markets may lead to market power, with an empirical analysis. The existence of incentives to exercise market power in energy trading makes the use of non-cooperative game theory problematic, as allocation systems like Shapley value can lead to incentives to increase prices in the energy market. This has led to different transmission pricing approaches in different countries. In [8], Richard Green draws comparisons of electricity transmission pricing in several countries according to six criteria that prices should satisfy.

In response to dissatisfaction with the current cost allocation scheme in New Zealand, a number of alternative proposals have been mooted. In this study we consider one recent candidate pricing scheme proposed by the NZIER [5]. This proposal is a variation on the *flow-gate* scheme for transmission rights (see [2]). Under the NZIER scheme, generators bid for the use of capacity on the HVDC in a rights auction. Given an allocation of capacity rights, the power transfer of the generator from the South Island to the North Island is restricted by its allocation. Secondary trading of allocation rights is proposed to allow transfer of these to generators who value them the highest, and a spot market is also proposed in which small quantities of transmission rights can be traded at the margin at the same time as energy is dispatched by Transpower.

In this report we examine the NZIER scheme from several perspectives. We show by example that a fixed allocation of transmission rights can result in an inefficient energy dispatch, demonstrating the need for a real-time balancing market in these instruments. We then study a stylized auction mechanism for allocating transmission rights by constructing a supply-function equilibrium for a system with two generators. The equilibrium model assumes that agents offer their generation at marginal cost in the energy market, but strategically offer demand functions for HVDC capacity that limits their output. The computational results from this model show that exercise of market power in the capacity auction leads to inefficient dispatch, and non-competitive prices in the energy market.

The report is laid out as follows. First, we will give a simplified modeling of the electricity market that takes into account the effects of the HVDC inter-island line. Then, we will introduce the game problem in electricity and study the case of no-line charge in chapter 3. In chapters 4,5,6 and 7, we actually study the proposed pricing scheme. First, we present it and examine briefly fairness and efficiency. Then, we consider the game on announced costs with simple rights allocation. Chapters 6 and 7 focus on line rights bids with a fixed and a uncertain demand. Chapter 8 briefly discusses an alternative line allocation. Finally, I give a statement about my internship before concluding in chapter 10.

Chapter 2

Modeling electric transmission

In order to have a better understanding of the consequences of an allocation of transmission rights, we need to describe the properties of electric transmission. There are two aspects. The first one is physical, and leads to a number of constraints. The second one is economics, and is controlled by the System Operator through an objective function.

2.1 Description of electric transmission and of the System Operator's process

Electricity has several specific properties, that complicates greatly the problem of transmission.

1. The flows in the network follow the physical laws of Kirchhoff. (Kirchhoff contraints (K))
2. Generators and lines have a maximum capacity. (Capacity constraints (C), divided into Generator Capacity constraints (C_G) and Line Capacity constraints (C_L))
3. The loss in transmission is a convex increasing function of the transfered power. (Loss constraints (L))
4. No power can be stored, thus the generation must be synchronised with the consumption, so that supply may meet demand at every node and at any time. (Demand constraints (D))
5. Consumers highly value electricity, but their demand vary through time.

The System Operator maximizes global efficiency. Hence the suppliers must be chosen among the most profitable generators able to meet the demand, according to the physical constraints. In order to do that, the System Operator runs a Linear Program that takes into account the characteristics of electric transmission and minimizes the global cost of the power generation.

2.2 Linear Program

We are now going to simplify the constraints, so that we can translate them into simple equations. In particular, we will try to get these equations as linear as possible. This will lead to approximations that we will discuss.

2.2.1 No-loss assumption

What happens when the losses on lines are taken into account is that there are geographical overcosts for generators distant from the demand. This is as if a tax has been added on those generators, or if they had an incremental cost of production. This caracteristics highly complicates the shape of solutions of the linear program. Therefore, we neglect them, so that equations are easier to write and results easier to present.

2.2.2 Network Modeling

We will be representing the transmission line network by a non-oriented graph. Each node i has a demand d_i. As consumers highly value electricity, we will make the approximation that the demand is inelastic, *i.e.* independent from the price. But it may vary through time.

Each node has an output x_i, which can be constrained to be nil, if there is no generator. But, it has to be positive. Moreover, through each line (i, j), a relative power $f_{i \to j}$ can be transfered from i to j, and it is the opposite of $f_{j \to i}$.

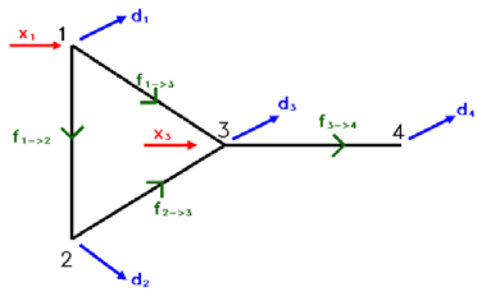

Figure 2.1: Example of a network

2.2.3 Kirchhoff's laws

In addition to flow conservation constraint is another one due to Kirchhoff's laws, called the loop flow constraint. If $X(i, j)$ is the reactance of a line, then the angles θ_i and θ_j in i and j will be related through the commonly used approximation $X(i, j) f_{i \to j} = \theta_i - \theta_j$, known as the DC power flow equation.

These equations can also be written as $Bf = 0$, where each line of the matrix B refers to a loop of the graph. Simply notice that it only affects loops, and that it is a linear condition.

2.2.4 Capacity constraints

Generator capacity constraints can be written $(C_G) : \forall \, \text{node} \, i, \; 0 \leq x_i \leq Q_i$, with $Q_i = 0$ if there is no generator at node i. Otherwise Q_i is the maximum capacity of the generators at node i.
Line capacity constraint can be written $(C_L) : \forall \, \text{link} \, (i, j), \; -U(i, j) \leq f_{i \to j} \leq U(i, j)$ where $U(i, j)$ is the maximum capacity of the line.

2.2.5 Demand constraints

Since supply equals demand, we have the demand constraint $(D) : \forall i, x_i + \sum_{(j,i) \in E} f_{i \to j} \geq d_i$. That means that the demand at node i can be supplied by the plant at the same point and the flows coming from its neighbour nodes.

2.2.6 Model

Let's sum up the remarks made so far.

Definition 1. *We define the following constraints.*

- $(D) : \forall i, x_i + \sum_{(j,i) \in E} f_{i \to j} \geq d_i$.
- $(K) : \forall (i, j), f_{i \to j} + f_{j \to i} = 0, \; Bf = 0$ *and* $x \geq 0$.
- $(C_G) : \forall i, x_i \leq Q_i$.
- $(C_L) : \forall (i, j), -U(i, j) \leq f_{i \to j} \leq U(i, j)$.

Notice that these constraints are all linear. We will now note $(DK) = (D) \cup (K)$, $(DKC_G) = (D) \cup (K) \cup (C_G)$ and $(DKC) = (D) \cup (K) \cup (C_G) \cup (C_L)$.

Now, every generator has a real marginal cost $\bar{c}_i$. However, it gives an announced marginal cost c_i (that might be different from $\bar{c}_i$) to the System Operator, which tries to maximize efficiency. In order to do that, it minimizes the total announced cost, that can be written $c \cdot x$. Thus,

Problem 1. *The System Operator chooses the dispatch x that minimizes total announced cost under constraints (DKC) with the use of the following linear program :*

$$(LP1) \; : \qquad C = \min_{(DKC)} c \cdot x$$

2.3 General properties

In order to have a better understanding of this process, and how it applies to the New Zealand network, we are now going to look into the mathematical properties of $(LP1)$ and some of its more simple versions.

2.3.1 The set of admissible points

Theorem 1. *If the graph of the network is not connected, then $(LP1)$ can be divided into independant similar linear programs that each refer to a connected component. This is true under (DK) with any of the other previously written constraints.*

This is due to the fact that constraint equations only link neighbour nodes. Therefore, from now on, we will suppose that the graph of the network is connected. We then have the following theorem.

Theorem 2. *Suppose (DK). Then any plant can supply any demand without excess. Moreover, there is then one single admissible flow.*

Proof. Suppose the demand is only at one point. Then Kirchhoff's laws say that, as you inject the same amount of power as the demand requests, the current will flow through the network in a determined way. Thus, the theorem is true if demand is located at one single node. But the equations are all linear. Therefore, by superposition, the theorem is proven for any demand. □

2.3.2 Spot prices

One important aspect of the electricity pool market is that not only does the linear optimization program give the dispatches of each plant, but it also sets the electricity spot price at each node.

Definition 2. *Dual variables of constraints (D) are the spot prices π.*

As a matter of fact, these dual variables represent how much electricity is valued. π_i represents the increase of the total cost, when demand at node i has an increase of 1 unit. Hence, a generator at node i that propose a production of electricity of x_i is selling a product that is valued at $\pi_i x_i$.

2.3.3 Structure of the solutions

We can now have a global understanding of what happens with (DKC_G) constraints.

Theorem 3. *Suppose (DKC_G) and the components of (c_i) are strictly increasing. Then x^* is the optimal solution to $(LP1)$ if and only if it satisfies*

$$\exists i, \forall j, \begin{cases} j < i \Rightarrow x_j^* = Q_j \\ x_i^* = \sum d_k - \sum\limits_{j<i} Q_j \\ j > i \Rightarrow x_j^* = 0 \end{cases}$$

In addition, spot prices are uniform and are equal to c_i.

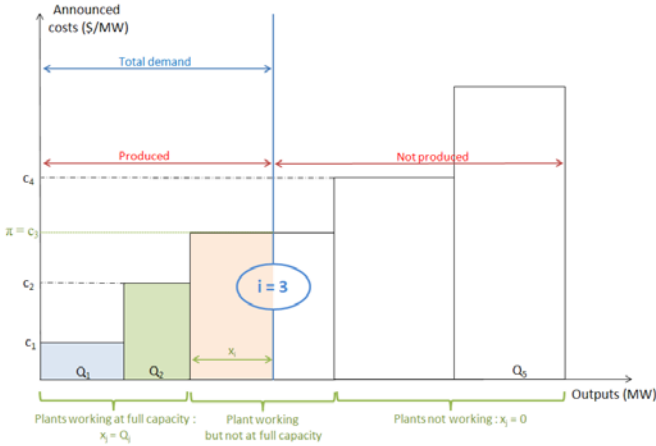

Figure 2.2: (DKC_G) dispatches

Proof. Figure 2.2 illustrates this theorem and gives a better understanding. Basically, as any plant can supply any demand, we just need to inject in the network as much power as the demand requires. Then, to minimize global cost, the cheapest plants need to be the supplier.

Now, if the demand changes a little bit at any node j, only that generator at node i will modify its production. Hence, the total cost of the production will increase with a coefficient c_i, which, thus, is the spot price at node j. □

Figure 2.2 can be interpretated as a supply function, that needs to meet the inelastic demand. The colored area is the global cost. Efficiency corresponds to the minimization of this area.

2.4 Line capacity constraints

Before dealing with the HVDC line, let us notice the major consequences that line capacity constraints can have with the following example. The point of this section is mainly to justify the assumption that no congestion exists inside each island network.

In order to do that, we will give us an example where line congestions play a major role.

2.4.1 Example

Consider the following network.

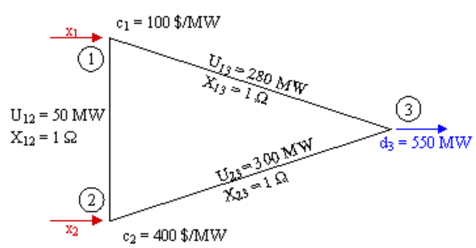

Figure 2.3: Example where (C_L) are critical

Here, we only have two generators 1 and 2, and all the demand is located at node 3. We suppose that the generators do not have any capacity limit, but lines do.

2.4.2 Set of admissible outputs

Figure 2.4 draws the constraints and locates the set of admissible outputs.

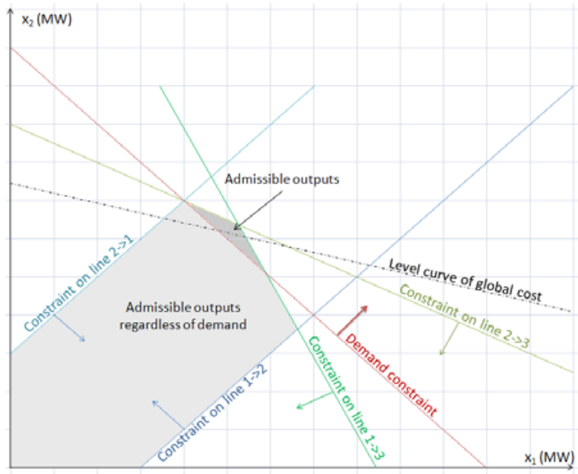

Figure 2.4: Constraints of the example on figure 4

2.4.3 Consequences of the line capacity constraints

If there were no line capacity constraints, then the set of admissible outputs regardless of the demand would be the whole top right hand corner of the plan. If you only add generator capacity constraints, then the set would be a rectangle. In both cases, it would be quite easy to find the lowest global cost level curve that intersects demand in the set of admissible points, since it would correspond to maximize the output of the low cost plant, and complete with the other plant.

In the case of line capacity constraints, we can have weird phenomena like here, where one generator needs the other to produce power, in order to increase its output. In this case, the demand can only be met if both generators work. Not only does it complicate the dispatch process, but it could also cause a power outage, if one of the generators breaks down.

2.4.4 Spot price increase

The spot prices in this example are very high. The optimal solution here can be read on the graph as the intersection of constraint $1 \rightarrow 3$ line and demand constraint line, which gives us $x_1 = 290$, $x_2 = 260$. Now, if the demand changes from 550 to $550 + \epsilon$, with ϵ very small, then the optimal solution will remain the intersection of those two lines. It will be $x_1 = 290 - \epsilon$, $x_2 = 260 + 2\epsilon$. Hence, global cost will be raised by $-100\epsilon + 2 \times 400\epsilon = 700\epsilon$. This means that spot price at node 3 is 700\$/MW, which is much much higher than announced marginal costs. Note that the spot price at node 1 (resp. 2) is the marginal cost of plant 1 (resp. 2), as we use the previous reasoning by considering the excess of outputs at generator nodes.

The reason of the high spot price is that, in order to have a higher global output, generator 1 needs to produce less, and generator 2 needs, not only to produce the increment that demand has asked for, but also what generator 1 has not produced in this process. Yet, marginal cost of generator 2 is much higher, hence the spot price.

2.4.5 Conclusion about line capacity constraints

When a congestion occurs too often, the electric transmission company reacts and increases line capacities. Solutions using line switches have also been considered. However, this is not the point of this paper. We will assume that there is no such problem inside each island network. The only congestion we will be facing concerns the inter-island HVDC line.

2.5 The inter-island HVDC line

As said earlier, the New Zealand transmission network has the particularity to be composed of two parts linked by a single line.

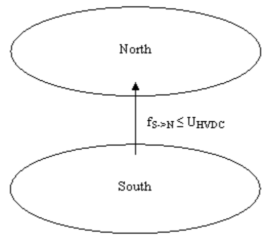

Figure 2.5: Inter-island HVDC flow

Moreover, costs of energy in the North Island are higher, whereas demand is also higher. Thus, the inter-island line is often congested by a flow that goes from south to north. Hence, our previously enunciated theorems cannot be applied directly. However, we will add a hypothesis, that will help us get results.

Definition 3. *In the rest of this paper, we will assume that the inter-island HVDC line is the only line that can be congested. We will note that constraint* (C_{HVDC})*, that, added to* (DKC_G)*, leads to* (DKC_GC_{HVDC})*. The flow through the HVDC line from south to north is* $f_{S\to N}$*, and has to have an absolute value lesser than HVDC capcity* U_{HVDC}*.*

2.5.1 HVDC line related notations

HVDC allocation proposals often try to compare what happens with what would happen, had the line not been there. The idea is that the difference between the case with and the case without the line, is the impact of the line. Those who take advantage of this difference should therefore be the ones who pay for the line.

As we will often compare those two cases, we will introduce the following notation. Variables that refer to the case without the line will now be noted with a 0 in superscript, which gives us x^0, π^0 or f^0 for instance, as opposed to the case with the line, with no annotation(x, π, f).

2.5.2 Results concerning the HVDC line

Now, with the previously defined hypothesis, we have the following result.

Theorem 4. *Suppose* (DKC_GC_{HVDC})*, dispatches are optimal, and flow goes from South to North. Then*

- *Southern (resp. northern) dispatches are the same as with the HVDC line cut and a demand incremented (resp. decremented) by the South to North power flow.*
- *Prices in South (resp. in North) are uniform.*
- *Southern price is lower than northern price (equal if there is no congestion).*
- *Southern price is higher than without HVDC line, whereas northern price is lower.*

Proof. Suppose $(LP1)$ gives a solution with a flow $f^1_{S\to N} \geq 0$. Then $(LP1)$has the same optimal solution if you add the constraint $f_{S\to N} = f^1_{S\to N}$, since we are reducing the set of admissible points, but keeping the optimum. Then, we can separate the linear program into two linear programs for each island, as the flow can be considered as a negative demand in the North Island. The optimal solution gives admissible points for the two programs, hence the existence of solutions.

For each island, we can apply theorem 3. This proves the uniformity of the prices in each island and gives us i^N and i^S, plants that are working, but not at full capacity, in the north and the South Island. Denote the northern price by c_{i^N}, and the southern price by c_{i^S}. If $c_{i^N} < c_{i^S}$, then i^N should produce the output of i^S, as it would minimize global cost. The flow it would generate is admissible, as it adds a north to south flow. But that would be absurd, since we have supposed the dispatches optimal. Hence the theorem. What's more, outputs in south increase, thus prices too. It is the opposite in the North Island. □

This theorem shows that southern generators take advantage of this line, as they can increase their outputs by exporting power to the North Island and benefit the southern price increase. On the other side, northern consumers also take advantage of the line, as their spot prices decrease. Accordingly, we will assume that the former plants export as much as possible to the North Island.

2.5.3 HVDC line congestion constraint

This leads to the following hypothesis.

Definition 4. *In the rest of the paper we will assume the HVDC line is congested. Added to the* (C_{HVDC}) *hypothesis, this gives us constraint* $(C_{HVDC}^{congested})$.

With $(DKC_G C_{HVDC}^{congested})$ hypothesis, we can now apply theorem 4 with the knowledge of the South to North flow. Hence, the problem can now easily be solved. Figure 2.6 illustrates the solutions.

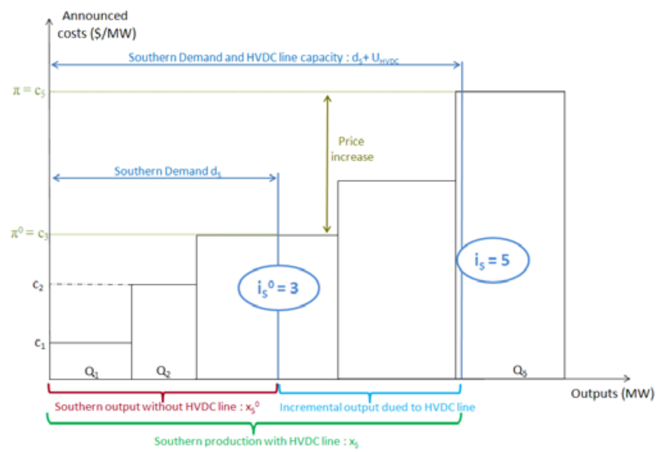

Figure 2.6: Incremental outputs in the South Island

2.5.4 Effects of the HVDC line

Despite losses in this line, global minimized costs get lower thanks to the inter-island HVDC line, which means that the dispatch is more efficient. Yet, as the cost of the inter-island is quite high (NZ$88 million per year), there comes the question of the allocation of this cost.

Obviously, we would like to have those using the line paying. However, it is physically impossible to allocate the part of the power of flow to each generator. Flows are mixed and have become indistinguishable. Therefore, some economists have proposed to compare outputs with the line, and without the line. The difference of outputs of each company should then be linked with the HVDC line allocation. The following theorem justifies this process.

Theorem 5. *Suppose there is no loss. There can be capacity constraints. Assume that* $(LP1)$ *gives an optimal solution where the flow through inter-island HVDC line goes from south to north. Then the total output of the North Island has now a lesser value than in the optimal solution of the problem where this line has been cut.*

Proof. - We can separate the linear program into two linear programs for each island, as in the proof of theorem 4. Therefore we have added a set of points for which the constraints are (K) and $\forall i, d_i' \leq x_i + \sum f_{j \to i} \leq d_i$. But, as we sum these latter constraints, we can see that the set of incremental points we are looking at lead to a total output lesser than the total demand of the North Island, which is lesser than the total output of the North Island with the HVDC line cut. Therefore, the optimal solution needs to be the solution with the line cut, or a point with a lesser total output, hence the theorem. □

Chapter 3

Introduction to electricity game

Now that we have a better understanding of the electricity pool market, we can focus on the main topic of the paper. In this chapter, we will describe a model of the market, and study the behaviour of its agents in a basic case, where there is no line charge.

Note that article [1] proves the existence of symmetric supply-function equilibria in an electricity pool market.

3.1 Generalities

Let us describe the main characteristics of the game problem.

3.1.1 Econcomical agents

So far, we have looked at the process used by the System Operator. However, in this economics problem, it is important to understand the behaviour of every actor.

The System Operator wants to minimize global cost. In order to do that, it has designed a dispatch process, that takes into account the announced costs of the generators. This problem, solved by $(LP1)$ will not be discussed.

Yet, it also has to design a transmission allocation process, which will affect the revenues of the companies. Given this process, companies have optimal strategies to maximize their profits. These strategies are what we will be studying.

3.1.2 Profits

In order to do that, let's have a look at the other agents of this system, the generator companies. On the supply function graph, we can read the profits made by each generator. Figure 3.1 illustrates this.

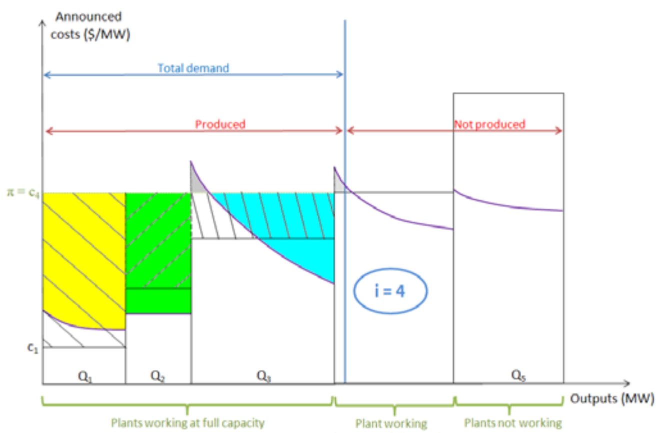

Figure 3.1: Profits

The profits that the System Operator can attribute to each plant is the hatched part. It is nil for plants 3,4 and 5. However, the real profits are the parts in bright colors, with grey parts corresponding to negative profits. These areas are located between real marginal costs curve and clearing price. Here, we have assumed that the real marginal costs $\bar{c}_i$ were decreasing, because of the economies of scale, and that there are now fixed costs. Of course, this latter hypothesis is quite debatable. We will try not to forget those fixed costs when discussing the results of our study.

In this example, the profit of plant 4 is negative. This is due to the fact that its announced cost is lower than real marginal cost. Yet, it does not mean that its strategy has been wrong. As a matter of fact, demand will vary through time. When it turns out to be high enough, so that plant 5 starts to work, it will have quite a positive profit, that it may not have had, had it announced a cost higher than that of plant 5.

3.1.3 Real costs and announced costs

Notice that the real global cost is different from the announced global cost that the System Operator tries to minimize. The real global cost is actually located under real marginal cost functions, on the left of the demand line.

Therefore, as announced costs may be quite different from real costs, it is an interesting question to find out whether, by minimizing announced global cost, the System Operator will be close to the real minimized cost allocation.

3.1.4 Cost function hypothesis

As we can see here, considering marginal costs $\bar{c}_i$ are not constant leads to a much more complicated game problem. Therefore, we will suppose these values constant.

We can have another explanation for such an approximation. As the number of plants is high, and as there is only one working-but-not-at-full-capacity plant, most of the time, plants are either not working, either working at full capacity. Then, supposing that we restrict ourselves to those cases, replacing the real marginal cost function of plant i by a constant marginal cost equal to the mean cost won't affect the game problem, as profits will remain identical (equal to $\pi Q - \int \bar{c}$ for a quantity Q at price π, with marginal cost $\bar{c}$). Either way, we have the following assumption.

Definition 5. *From now, on we will assume that fixed costs are nil and real marginal costs are constant. This assumption will be noted* (H_{cost}).

3.1.5 Game problem

In practice, all the plants are owned by a small number of companies. Now, each company k possesses a set I_k of plants. Their goal is to maximize the total flashy parts above their plants, given the rules of the dispatch and the electric transmission network allocation, by setting the right announced costs. In our case, dispatch rules are following theorem 4 and figure 2.6, since we will be making assumption ($DKC_G C_{HVDC}^{congested}$), and the electric transmission network allocation is restricted to the HVDC line allocation, in the forms described in the next sections.

We have almost written the problem as a game theory problem. Each company is a player, that will be regarded as a rational and intelligent person. Notice that this game happens every 30-minute, for which an estimation of the demand can be made by all the players. Yet, we will suppose that there is an uncertainty about this demand. Given that, the players have to choose their announced costs (and, in the pro-rata-with-incremental-output allocation, their line rights bids). Therefore, we will be looking at profits over a long period of time.

In the next chapters of this paper, we will mainly consider a pro-rata-with-incremental-output allocation for the inter-island HVDC line. We will try to solve the game problem, and look at the announced costs that the players will be choosing, as they maximize their profit. In particular, we will study the existence and the stability of Nash equilibria.

3.2 Game with no line rights charge and no line congestion

In this section, we will focus on the simplest case. This example is fundamental, because it reflects what happens when no line is congested, and there is no rights allocation for lines. Nevertheless it is still quite complicated.

3.2.1 Hypotheses

The problem here is a game problem, where, given a marginal cost function and the System Operator's rule of dispatch, companies maximize their profits by choosing announced costs for their generators. We will suppose that the number of plants is high enough to approach company real marginal cost and announced supply curves with increasing, continuous and regular functions.

Also, since there is no line congestion, the rule of dispatch is to choose the rights spot price, so that any generator whose announced cost is less than the price works at full capacity, and so that their cumulated output is equal to demand.

Now, we suppose that the companies choose their announced cost curves simultaneously, and there is uncertainty about the demand. Our goal is to highlight a Nash Equilibrium.

3.2.2 Company viewpoint

Every company k has a real marginal cost function $\bar{c}_k(q_k)$, where q_k is the output, and looks for the optimal solution c_k^*.

It supposes that the cumulated cost function of its rivals is $C_{-k}(q_{-k})$. The link between this function and the cost functions of the rivals is that, for any price p, $C_{-k}^{-1}(p) = \sum_{j \neq k} c_j^{-1}(p)$ (the quantity offered at price p by the set of rivals is the sum of the quantities they each offer).

When the demand d is fixed, given the cumulated cost function of its rivals, a company k simply has to choose a couple (q_k, c_k) so that $c_k = C_{-k}(d - q_k)$ and so that the company's profit is maximal. As demand varies, we get a curve $c_k(q_k)$.

The following figure illustrates the choice of these points.

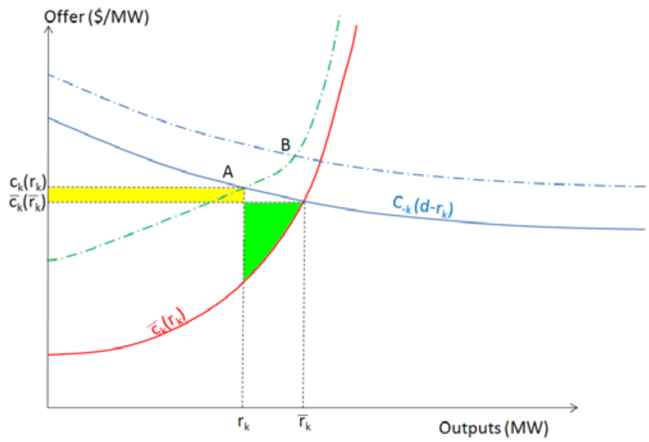

Figure 3.2: Best reply curve to other company's supply function

The condition $c_k = C_{-k}(d - q_k)$ means that the company k has to choose a point on the curve. The maximization of the profit condition means that the yellow area is equal to the green area. Hence, we get the point A. For another demand, we get the point B. As demand varies, we get the green curve, which is the best reply of company k to the bids of its rivals. We may have better results using the Z function introduced in 7.1.

Note that the more the blue curve is horizontal and the red is vertical, the more the equilibrium is close to the marginal cost curve. Yet, the more company there is, the more the blue curve will be horizontal.

3.2.3 Equilibrium

Now, for any given set of supply functions $c = (c_k)$, there is a best reply $BR(c) = (BR(c)_k)$. The existence of a Nash Equilibrium is equivalent of the condition $\exists c, c = BR(c)$.

A common technique would be to look at the the sequence (c_n) defined by $c_{n+1} = BR(c_n)$, with for instance $c_0 = \bar{c}$ or $c_0 = \bar{c} + c^0$, where c^0 is a constant.

If there is a solution, especially when the number of companies is high, we can note that efficiency would not be that bad, as every company has an announced cost slightly higher than the real marginal cost.

Moreover, the more companies there is, the more we may assume the cumulated costs of other companies more horizontal, and the closer the announced costs will be from real marginal costs.

Chapter 4

The pro-rata-with-incremental-output allocation

Right now, in June 2010, southern generator companies pay a tax pro rata with their maximum production in the South Island. In [7], market power is enhanced by considering the game generated by the electricity pool market with financial transmission rights. But other market designs have been suggested. One of the most important suggestion is a pro-rata-with-incremental-output allocation. We are going to introduce it in this chapter. Chapters 5, 6 and 7 also consider this allocation.

4.1 Description of the allocation

Rights for the line could be sold. Any company k would then be allowed to produce the quantity r_k related to this right in addition to what they would have produced, had the line not been there. Thus there would be another market, the market for the rights to the line, that would be decomposed into three phases.

First, an auction or a pro-rata-with-previous-years-use system, or a mixed system, would define the initial allocation of the capacity rights. Then, every half-hour, secondary trading would enable Transpower to offer the rights, that have not been allocated in the first place. Finally, generator companies would enter every half-hour a spot trading of capacity rights, where they would bid and offer rights. This would lead to a clearing price, and to the transactions.

We can note that the spot market for rights trades is equivalent to a system where rights are rented for every half hour. Then companies would make bids, and a clearing price can be chosen so that the sum of the rights reaches the total amount of rights.

4.2 Unfairness

The question of the fairness of the allocation is important. Since we want the energy market to be as liberal as possible, so that anyone who thinks he can produce energy more efficiently can easily enter the market. Furthermore, if the allocation is too unfair, complaints will quickly rise, and the market design would have to be changed.

Basically, we want the people who take advantage of the HVDC line to pay for the use of it. Accordingly, this idea of a pro-rata-with-incremental-output allocation has been suggested. However, as we will see in this section, this allocation is not fair.

Suppose 5 companies each possess one plant. We are working under $(DKC_GC_{HVDC}^{congested}H_{cost})$ hypotheses. Let's assume that the price of the use of the HVDC line is p_{HVDC} for the incremental outputs. Then we can draw the following graph (figure 4.1).

Let us well distinguish the profits made with the HVDC line cut, in dark colors, and the profits made by virtue of the HVDC line, in bright colors. The parts in grey correspond to the HVDC line cost contributions, and can be count as a negative profit for the company in which the grey rectangle is.

We can see that every company makes incremental profit, thanks to the HVDC line, because the increase of the global output has lead to an increase of the southern spot price. Moreover, it is interesting to notice that the companies who make the most profits thanks to the line are not exactly the same as the companies who produce more thanks to

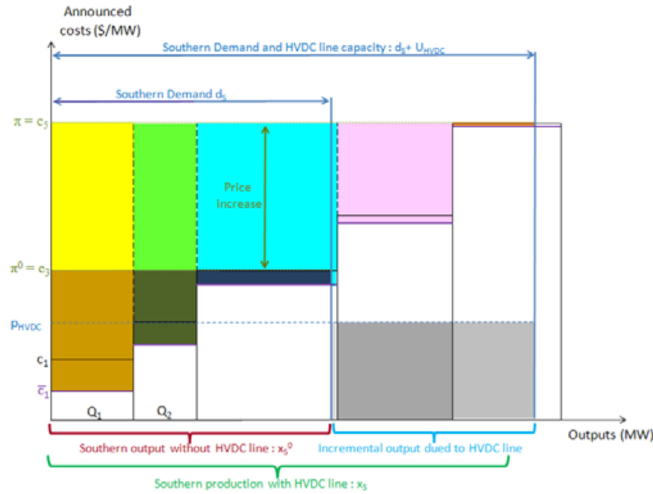

Figure 4.1: Profits in a pro rata with incremental output allocation

the line. As a matter of fact, generators 1,2 and 3 all make a large incremental profit, but plant 5 does not make an important incremental profit.

Yet, despite its relatively low profit, plant 5 is one of those who pay the most for the use of the line, which leads it to a negative total profit, if we take into account the line allocation. On the other hand, plant 1,2 and 3 do not pay a single dollar and totally take advantage of the line.

That's why this pro rata with incremental output allocation is unfair. Relatively expensive plants of the South Island are hugely disadvantaged by this allocation. An alternative scheme that we call pro-rata-with-incremental-profit allocation is described in chapter 8.

4.3 Inefficiency without line rights trades

In this section, we will suppose that companies possess rights on the line that cannot be sold, or that will not be sold. The reason can be individual interests, erroneous reckonings, lack of time to exchange rights or the inexistence of a spot market for rights. What we are now going to look at is the inefficiency that the line allocation can cause.

Suppose we have 3 companies. Companies 1 and 2 each possess 2 plants, whereas company 3 possess another plant. Each company has about one third of the total rights on the HVDC line. We are working under $(DKC_GC_{HVDC}^{congested}H_{cost})$ hypotheses. The following graph shows how the dispatch will be chosen.

The pink and the green areas are the costs of incremental outputs in an efficient dispatch. However, because each company has rights to produce the output that would have made with the HVDC line cut, incremented by a quantity pro rata with the value of their rights, they all have incremental outputs. Therefore, plant 4 and 5 of companies 3 and 1 are now working. The real incremental costs are in pink, yellow and blue.

The pink area is the common incremental costs. The yellow area represents the difference between the incremental costs when companies possess rights and the incremental costs in an efficient dispatch. As we can see here, this area is quite important, which shows that this allocation system is quite inefficient.

What's more, the price with this allocation is higher than the price in an efficient dispatch, which is a major drawback for the consumers. In fact, we have the following theorem.

Theorem 6. *Suppose that the real and announced costs are all different, and that real costs are ordered in the same way as announced costs. Then, there exists a unique rights allocation that leads to efficiency.*

Proof. This allocation corresponds to the incremental outputs between efficient dispatches with and without the HVDC line. Since this allocation gives efficiency, the existence has been proved. Now, suppose we have another allocation, then that means that there is at least one plant that is not working as much as in an efficient dispatch. Instead, a strictly more expensive plant has rights to dispatch and is producing. Give some of these rights to the former plant, and efficiency would be strictly improved, thus the unicity. □

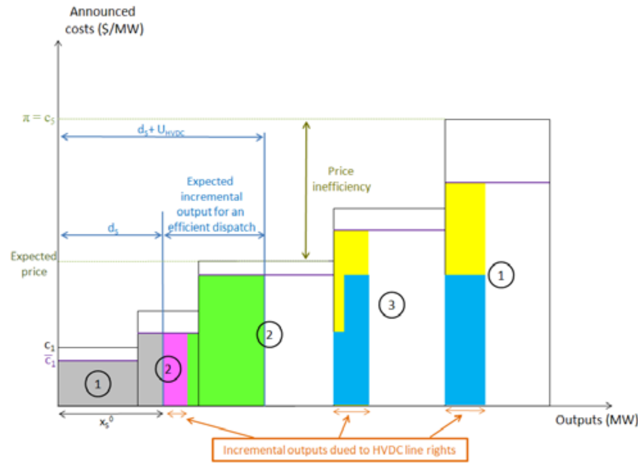

Figure 4.2: Inefficiency in a pro rata with incremental output allocation

If the announced costs are not all different, existence can still be proven. But, there can be a set of optimal allocations. This set would be a polyhedron.

Chapter 5

Game on announced costs

We still consider the pro-rata-with-incremental-output allocation.

So far, we have shown inefficiency and unfairness with given announced costs and rights allocation. However, those variables are actually chosen by the companies. Therefore, as these companies want to maximize their profits, they do not choose those variables randomly. They are faced with a complex game problem, that we are now going to study.

In this chapter, the game on rights allocation will be reduced to its basics. What will interest us is the kind of electricity supply functions that companies may be tempted to bid.

5.1 Asymmetric duopoly example

In order to understand the game on announced costs, we will take a simple example of a duopoly. Our purpose here is to illustrate how one player in the duopoly can have an increased profit by bidding strategically, but not to compute an equilibrium.

5.1.1 Preliminary remarks

Let us notice that if their announced costs are higher than the northern price, then southern generators which are supposed to supply incremental outputs will be replaced by northern generators whose announced costs are lower. Therefore, there is this limit for southern generator offers.

We can also notice the difference between two kinds of plants, those which enable profits with high outputs and low real costs and those which increase the southern spot price. Also notice that there only needs to be one company that possesses plants whose goal is to increase southern spot price.

5.1.2 Hypotheses

We are supposing that the companies have an estimate of the southern demand, and of the northern demand. With the northern demand, they can estimate the northern price. Also, we will suppose that there is a correlation between the southern demand and the northern demand, hence with the northern price, as people have the same reasons to increase power usage.

Suppose there are companies 1 and 2. Company 1's (resp. 2's) generators all have a real marginal cost equal to $\bar{c}_1$ (resp. $\bar{c}_2$). Assume there is no generator capacity constraint and that company 1 is the only strategic company, as the other one sets its announced costs $c_2 = \bar{c}_2$.

Suppose now that company 1 has an infinite number of generators, so that he can choose as many announced costs and capacity limit as it wants. This can be regarded as the choice of an increasing supply function $s(x)$, to meet any demand. The game problem for company 1 is now to find the rights supply function that will maximize his profit over the period corresponding to the length of his choice of announced costs.

5.1.3 Shape of the solution

Theorem 7. *The following bold line gives the shape of the solution of our problem (the value of Q_0 is the only degrees of liberty), with a purchase of all the line rights.*

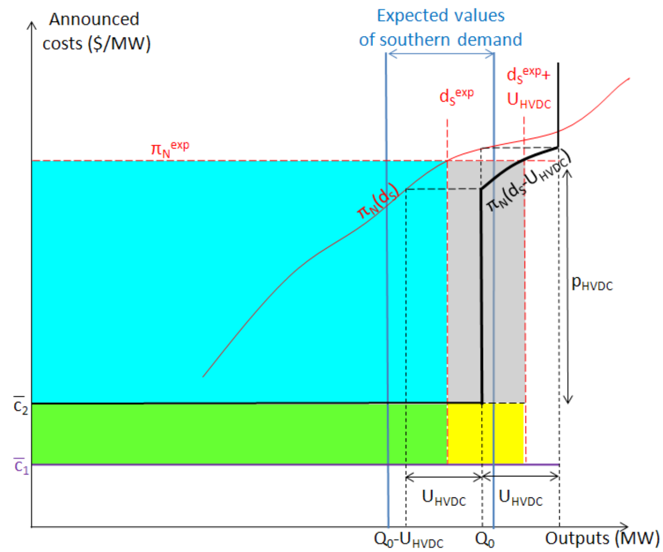

Figure 5.1: Supply function and strategies of company 1 in a pro rata with incremental output allocation

The green area represents company 1 profits, had there been no HVDC line. The blue area is his profit thanks to this supply and the increase of price. The yellow and grey part corresponds to incremental profits due to export, but the grey part is also the cost of the line rights. Therefore, total benefit is the sum of green, blue and yellow areas.

Proof. If c_1 is always less than c_2 then company 1 cannot win more than the green area, since the electricity price is $\max(c_1, c_2) \leq c_2$. The drawn black line with Q_0 equal to the highest expectable value of southern demand always does better (strictly better for high demands). Hence, in the optimal curve, there is a quantity for which $c_1 > c_2$. Call Q_0 the lowest quantity of the supply function for which $c_1(Q_0) \geq c_2$ (Q_0 may be nil).

Note that company 2 values line rights at $\pi_S - \bar{c}_2$, so company 1 needs to pay slightly more to have all the line rights at lowest line rights price. Hence, the green area will always be a rectangle whose bottom is c_2 and whose top is π_S.

For southern demand in $[Q_0 - U_{HVDC}, Q_0]$ (which can be always true if the width of the range of possible southern demand is less than the HVDC line capacity), the profit reached with the drawn line as supply function is the maximum reachable profit. As a matter of fact, the southern price, defined by the intersection of the supply function and the cumulated demand $d_S + U_{HVDC}$, is equal to the northern price, which is the maximum it can be. Moreover, the total output of company 1 is $d_S + U_{HVDC}$, which, once again is the highest output reachable by company 1.

Now, if the southern demand is less than $Q_0 - U_{HVDC}$, he can only hope for the green part, which will be maximized for c_1 slightly less than c_2. Therefore, company 1 should have a horizontal line before Q_0, slightly under c_2.

And if the southern demand is higher than Q_0, then his output will be limited to $Q_0 + U_{HVDC}$. The section of his bidding curve beyond this point won't matter. For instance, it can be vertical, which will make sure that the southern price always equal to the northern price. Company 2 will take advantage of that.

Hence figure 5.1 gives the shape of the optimal bidding curve, with a degrees of liberty on Q_0. □

5.1.4 Determination of Q_0

The problem is now for company 1 to choose the right Q_0. In order to do that, the company has now to look at its profits, described in the following figure.

The two compared profits are the red line and the blue line. Superpositions are in purple. In the red case, Q_2 is higher than in the blue case (Q_1). The pink area is the part that red line does better than blue line. The yellow area is the opposite. Let us notice that when the Q_1 and Q_2 both increase of the same amount, the pink area diminishes, whereas the yellow area increases. What's now interesting to calculate is the limits $l_{pink}(Q_0)$ and $l_{yellow}(Q_0)$ of the two areas divided by $Q_2 - Q_1$ when $Q_2 \to Q_1 = Q_0$.

Now, the southern demand varies during the year. We may assume that the distribution of it is roughly a Gaussian function. Let f be the distribution. Then, we are now looking for the Q_0 that maximizes $\int \Pi f$, where Π is the profit

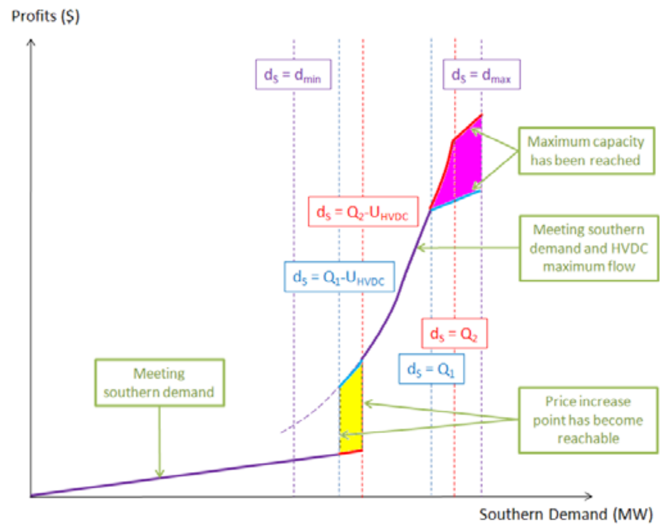

Figure 5.2: Comparison of profits of Company 1, with two values of Q_0

that depends on Q_0. This gives us a condition, that we will not write because of its length. Roughly, $l_{yellow}(Q_0)f(Q_0 - U_{HVDC}) = l_{pink} \int_{Q_0}^{d_{max}} f$. We said roughly, because the second term is not exactly that. As l_{yellow} is increasing and l_{pink} is decreasing, there is a unique solution to that problem.

5.1.5 Conclusion about the asymmetric duopoly example

In short, our study has shown, that company 1 has a strategic solution to maximize its profit. Obviously, since it concedes outputs to its more expensive rival, this strategy does not lead to efficiency.

5.2 Generalisation of the example

We are now going to try to generalize our observations.

5.2.1 Oligopoly

We can notice that this strategy of company 1 is the right one even if he has a small number of competitors in the South Island, as long as he is the one with the lowest real marginal costs, and as long as the other companies play non strategically by choosing an announced cost slightly higher than their real marginal costs.

5.2.2 Company 2's response

In the case where company cannot buy line rights, it may still have a better response than the dumm strategy. As a matter of fact, it could slightly undercut the company 1's announced cost. His profit will be positive whenever company 1 will manage to set a high price. This means that if company 2 undercuts at price $\bar{c}_2 - \epsilon$ for a quantity q_2, then his profit will be slightly negative when $d + U_{HVDC} \leq Q_0 +_2$, but will be quite positive when $d \geq Q_0 +_2 - U_{HVDC}$.

Now, if company 2 he can buy line rights as long as company 1 does not buy them, then another strategy is to buy all the rights as soon as the northern price is higher than $\bar{c}_2 + p_{HVDC}$, and set the southern price at the northern price, until company 1 buys the rights. Then, it can simply have a constant announced cost equal to the value it had when it gave the line rights away.

5.3 Coalition

The point of this section is not that much about forecasting the behaviour of the companies. The point is more about illustrating the incentives there are for players not to play in a competitive way.

5.3.1 Two player example

In order to do that, consider the example previously mentioned of two players with infinite capacity to illustrate coalitions.

If the two companies are having a coalition, then the first one could use the second one to reach the northern price. Company 2 supply function would be a vertical line for a nil production. With an infinitely small number of rights, it would set the southern price at the northern price and pay almost no line right.

The first company would then simply have an announced cost equal to his real marginal cost. Then he would always be producing the southern demand. If the northern price is higher than his real cost incremented by line rights price, then he would buy all of the line rights.

Since this will be the maximum profit the coalition can hope for, it is a Pareto equilibrium. They can then easily divide their gains so that every one gets more than in the previously described competition allocation.

5.3.2 In general

This is still true as the number of companies grow. One of them can still be used in the single purpose of increasing the southern price. Although it would then lead to a an efficient distribution, where the most profitable will be chosen to produce, the consumers of the South Island would be losing a lot, as the prices rise.

However, any company would be tempted to deviate from this Pareto situation, by bidding electricity at an nil announced cost.

It is important to notice that company 2 has simply sacrificed itself. Yet, if no existing company does what it does, it would be quite easy for existing companies to create another one that plays that part.

Of course, this would be quite easy to track down, and it is also possible that line rights purchases may be restricted to the three main companies. Still, with this allocation, there is a real threat to strategic non-competitive behaviours.

Chapter 6

Game on line rights with certain demand

We still consider the pro-rata-with-incremental-output allocation.

If this allocation was established, the distribution of line rights would be a major problem for the System Operator. Lately, it has been proposed that the allocation of them would rely on an auction. Hence, in addition to the electricity market, there comes a line rights market.

Those two highly-linked markets lead to a very complicated game problem, as the space of strategies becomes much larger. In this section, we limit ourselves to the game problem on the line rights market. As a matter of fact, we will assume that companies simply bids their real marginal costs in the electricity spot market.

6.1 Line rights spot market with a high fixed southern price and non-strategic players

What will be studied here is some kind of ideal line rights market.

6.1.1 Hypotheses

First, we are going to study a case where companies are non-strategic. The correct allocation prices correspond to how much companies really value their line rights, according to their announced costs. This means that they represent their marginal profits.

Companies have bought all of the line rights in the beginning of the year. Yet, now, they can exchange them, at the price that they settle themselves. These trades can happen every 30 minutes and follow the variations of the demand. This demand is certain and known by all the agents.

Now, it is actually even easier for companies to adjust their outputs and their rights, so that the northern price is as high as possible, thanks to a plant which has an actual output almost nil. That's why, from now on, we will suppose that the southern price π_S may not have reached the northern price, but is definitely higher than the other working plants. Another explanation of this would be the absence of market power, possibly due to a high number of competitors.

6.1.2 Line rights exchanges

Thus, as the southern price is fixed, the incremental profits due to a plant are pro rata with its outputs. Therefore, every plant just wants to produce, and the optimal announced costs are the real marginal costs.

Consequently, the values attributed to the line rights by a company are the marginal profit $\pi_S - \bar{c}_i$, where i is the company's cheapest plant not working at full capacity. Companies are ready to buy line rights if the clearing price of a line rights is less than $\pi_S - \bar{c}_i$, and they are willing to sell line rights if the clearing price is higher.

The supply (resp. demand) function can be obtained by plotting the aggregated marginal profits of plants which possess (resp. do not possess) the line rights. As the two curves are step function, they will probably not actually meet. Therefore, although the quantity of exchanged rights can easily be read, there is only a range of values of the clearing price that can be deducted by these functions.

Note, that given the clearing price, incremental profits due to line rights exchanges are the colored rectangles of the supply and demand graph.

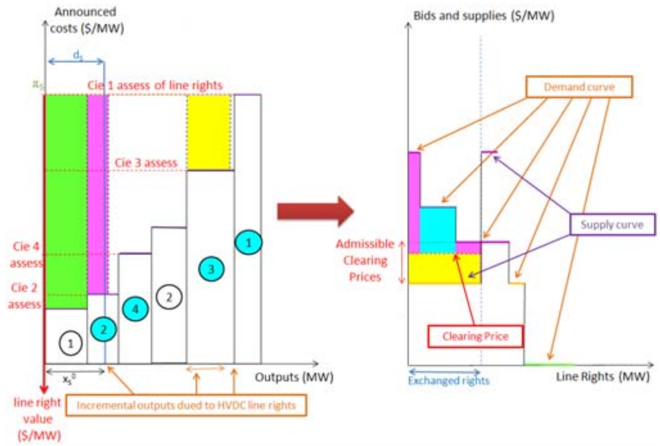

Figure 6.1: Supply and demand of line rights

6.1.3 Equivalent linear program

Let us observe that this line rights allocation is also the result of a linear program, as the following theorem states.

Theorem 8. *Suppose that the dispatch without line trades is x^0 and consider the following linear program :*

$$\max p \cdot x - p \cdot y$$

$$\text{Subject to : } \begin{cases} p = \pi_S - \bar{c} \\ \sum x_i = \sum y_j \\ x_i \geq 0; x_i + x_i^0 \leq Q_i \\ y_j \geq 0; y_j \leq x_i^0 \\ \forall \text{Cie } k, \sum\limits_{j \text{ of Cie } k} y_j \leq \text{ Rights of Cie } k \end{cases}$$

The optimal solution of the linear program gives the line rights purchases x, and the line rights sells y. The dual variable of the second constraint is in the range of admissible clearing prices.

Thanks to this linear program, negotiations in the spot market or transactions made by the System Operator can be done quickly and easily.

6.1.4 Result

Anyway, no matter how the clearing price is chosen among the admissible clearing prices, the exchange of line rights is the same and leads to the following figure.

The very dark areas are the incremental profits due to line rights exchange. Hatched areas are line rights expenditures, but also profits made, hence they count like nil.

Notice that the spot market of line rights leads to an almost efficient dispatch. Yet, recall that we have made the hypothesis that the southern price was fixed and high. Hence, this case is not an actual likely case.

6.2 General game with line capacity constraint and certain demand

In this section, we will study the line rights auction with strategic players. For a major part of this section, we will not make any assumption about the certainty on demand or the capacity of the HVDC line and try to remain general. Only subsections 6.2.4, 6.2.6 and 6.2.7 deal with HVDC line capacity.

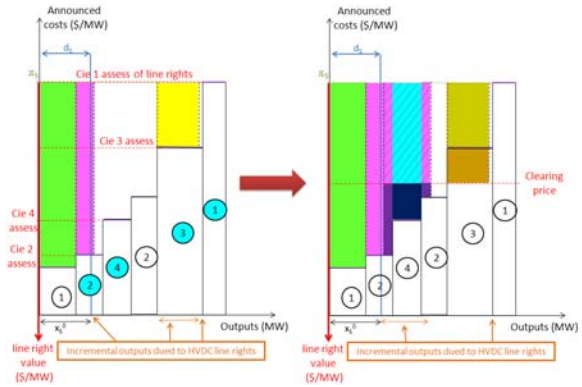

Figure 6.2: Line rights trades in a spot market

6.2.1 Hypotheses

Once again, we will assume that companies announce their real marginal costs. Moreover, in order to avoid problems of discontinuity, we will suppose that the number of plants is so high, that the marginal cost functions can be approached by a continuous and regular function.

As the dispatch for the South Island is now fixed, we will only consider the remaining cost functions after that dispatch. Each company has a marginal cost function $c_k(r_k)$. Its profits actually only depend on the price set by the other companies and its own output. There, it can be written $\Pi_k^p(r_k)$, where p is the cost set by the others and can be written $\max_{j \neq k} c_j$. We will note the marginal profit $\rho_k^p(r_k) = [\Pi_k^p]'(r_k)$.

Now, given these cost functions, each company has now to give bids for line rights, that will lead to a clearing price, and a line rights allocation.

6.2.2 Three main situations

The following graph shows a typical marginal cost function.

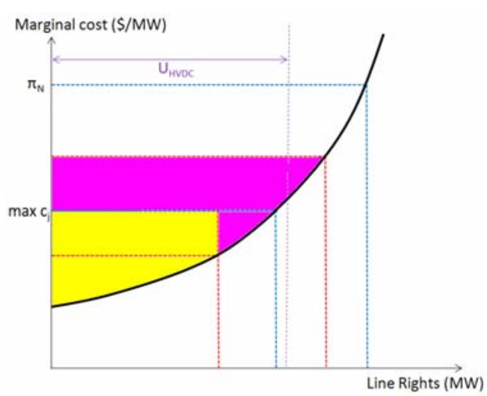

Figure 6.3: A typical marginal cost function

Examples are representated by the dotted red lines. Colored areas are profits. The spot price of the electricity in the South Island is always $\pi_S = \max_{\text{Cie } j} c_j(r_j)$, where j corresponds to companies of the South Island. It is important to differentiate 3 parts in this graph, delimited in dotted blue.

First, when $c_k(r_k) \leq \max_{j \neq k} c_j(r_j)$, the profits of company 1 are the bright yellow area. The spot price of the electricity is the price $\max_{j \neq k} c_j(r_j)$ set by the other companies. The marginal profit here is positive, since it is $\rho_k^{\max c_j} = \max_{j \neq k} c_j(r_j) - c_k(r_k)$.

Second, when $\max_{j\neq k} c_j(r_j) \leq c_k(r_k) \leq \pi_N$, company 1 is the one who actually sets the price. The profits are the yellow and pink areas. The marginal profit can be written $\rho_k^{\max c_j} = r_k c_k'(r_k)$, which is positive and increasing if c_k is convex.

Third, when $c_k(r_k) > \pi_N$, price can no longer increase, nor can the output. Line rights are useless, since marginal profit is nil. Profits are the area under price π_N and above the marginal cost curve. It is definitely a situation to avoid.

6.2.3 Profits

Let us now have a look at the profits. The following figure shows the profits for several values of the maximum marginal cost of the rivals, the blue being when $\max_{j\neq k} c_j = \pi_N$.

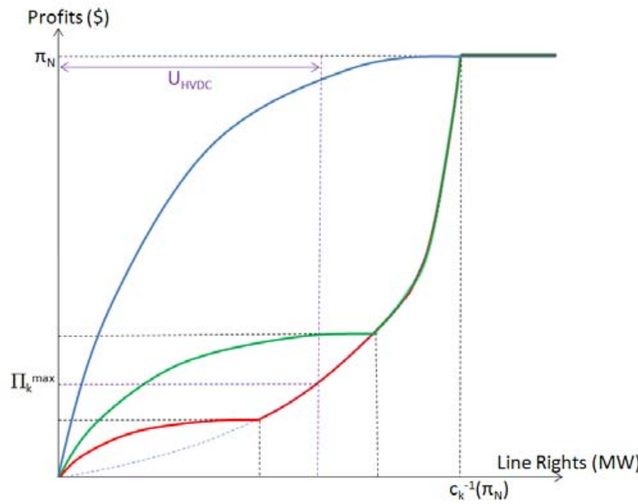

Figure 6.4: Profit curves

As we can see, the curve can be separated in the 3 sections previously described. Now, if the prices of the line rights were nil and if there were no constraint on the maximum capacity of the line, then the best reply of company k to the costs c_j of the rivals is the maximum of the curve $\Pi_k^{\max c_j}$. This maximum is reached at $r_k = c_k^{-1}(\pi_N)$.

Notice that when the price set by the rivals is π_N, like on the blue curve, the second section of the curve has disappeared.

6.2.4 Two main strategies

Observe a fundamental fact. Companies can either take advantage of the price set by their rivals, or they can set the price. The higher is the price set by the rivals, the more incentives the company will have to take advantage of it, instead of fighting for line rights. But if the price set by the rivals is low, it may be better to choose to set the price itself.

Now, if all the companies can reach their maximum profit without congesting the line, which means that $\sum c_k^{-1}(\pi_N) \leq U_{HVDC}$, then we have Nash Equilibria $r = (r_k)$ with a line price equal to zero when $\forall k, r_k \geq c_k^{-1}(\pi_N)$ and $\sum r_k \leq U_{HVDC}$.

However, we can assume that the number of players is high enough so that the second condition we mentionned cannot be met. Therefore, companies may fight for line rights, which will set it strictly positive. In fact, their marginal profit is how much they are willing to pay to have an incremental unit of line rights.

6.2.5 Marginal profits

Therefore, in order to have a better understanding of those strategies, we may have a look at the marginal profits. The corresponding figure is figure 6.5, where are drawn the marginal profits corresponding to the profits of figure 6.4.

Now, the discontinuities in this graph cause troubles for the allocation of the line rights. Basically, they can either bid for the first part of these curves, which means that they are counting on another company to increase the price, in

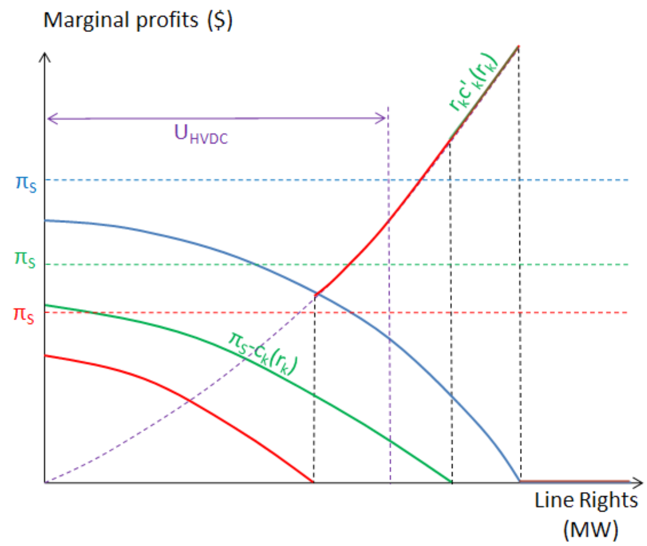

Figure 6.5: Marginal profit curves

which case they have a decreasing bid. Or they can be the one who set the price high, by the second part of the curve. In this case, they would directly bid for $c_k^{-1}(\pi_N)$, because the marginal profit is higher at this point.

6.2.6 First strategy

We are now going to assume that the northern price cannot be reached because of the line capacity. Let us consider a company k, and suppose it will bid the first part of the curves, which means that it counts on another company to set the price high. Then, it will have to maximize the orange area in the following figure.

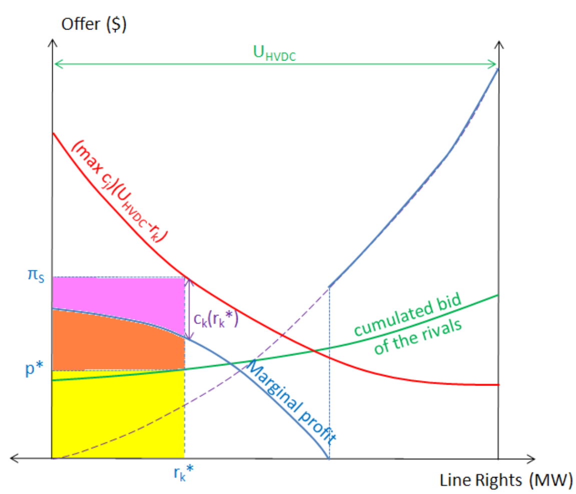

Figure 6.6: Best reply curve when taking advantage of a higher set price

The yellow area is the cost of the line, while the pink area is the cost of the output.

The output that company k chooses corresponds to a point on the cumulated bidding curve of its rivals (green curve), which gives the line price. It also corresponds to an electricity price set by the rivals (red curve). This electricity price defines the marginal profit curve of company k (blue curve). Eventually, we can read the actual profit of company k, since it is the orange area.

6.2.7 Second strategy

The company has another option, which is to bid for the second part of the curve. But, the increase of this curve means that he would rather buy all the line rights than some of them, no matter what the price is, as long as his profit remains positive. If it does not, then he would buy nothing. This limit price is $\pi_k^{lim} = c_k(U_{HVDC})U_{HVDC} - \int_0^{U_{HVDC}} c_k$.

Still, the System Operator may require that the bids are decreasing curves, in order to have a fair dispatch. The following graph shows the bid the company wishes it could announce (green curve), and the bid it would have to announce (red curve).

Figure 6.7: Optimal curves when fixing the price

Now, it is important to notice that either one company takes advantage of the price set by other companies and shares the line (sharing strategy S), or bid for the whole line rights at line price p while setting the price itself (monopolistic strategy $M(p)$).

6.2.8 Mixture of the two strategies

Notice that the first strategy plays on a low quantity and the second on a high quantity. We can imagine that companies mix these two strategies with a curve that, in the first part, plays the first strategy, and then plays the second strategy.

6.3 Symmetric duopoly example with line capacity constraint and certain demand

Let us illustrate that with a two player example, where marginal costs are identical linear functions $c(x) = cx$. Note that c is the second derivative of the total cost. For clarity of explanation, we will note $U_{HVDC} = U$.

6.3.1 Best response to a monopolistic strategy

If a company tries strategy $M(p)$, then its bidding curve is the red curve, where the maximum price of purchase p can be chosen in the range $[0, \frac{1}{2}cU]$. If it succeeds in buying the whole line, which happens when he plays against $M(p_0)$ with $p_0 < p$, then its profit will be $\Pi_{M(p)}^{max} = \frac{1}{2}cU^2 - pU$. If $p_0 = p$, then we will assume that the two companies share the total amount of rights and get $U/2$ each. Their profit will be $\Pi_{M(p)}(M(p)) = \frac{1}{8}cU^2 - \frac{1}{2}pU = \frac{U}{8}(cU - 4p)$.

However, a company can also guess that the other company will choose strategy $M(p)$ and bid the red curve. Hence, it may want to try to take advantage of the high set price. He would use strategy S, accordingly to figure 6.6. Therefore, he chooses a point (r, p) so that $(U - r, p)$ is on the red curve of the other company. Then, his profit can be written $\Pi_S(r, M(p)) = (cU - p)r - \frac{3}{2}cr^2$. The maximum of this function is reached when $r = \frac{cU - p}{3c}$, which means that $p = c(U - 3r)$. This is the bidding curve of strategy S. His profit, as a function of p is now $\Pi_S(M(p)) = \frac{1}{6}(cU - p)^2$.

The following curve now shows the profits of strategies S, $M(p_1)$ and $M(p_2)$ (with $p_1 < p_2$) when playing against $M(p_0)$.

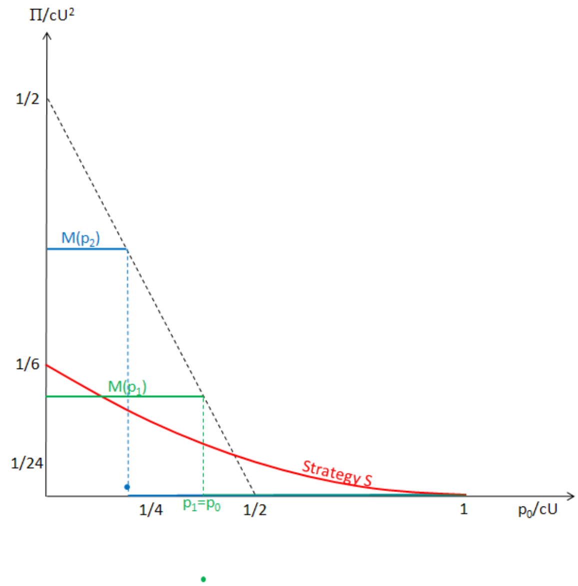

Figure 6.8: Profits of different strategies against $M(p_0)$

6.3.2 Best response to a sharing strategy

Now, let us have a look at the best replies to strategy S. If the other company plays S, then the first company needs to choose a point of the bidding curve such that its profit will then be maximized. Since it only has to choose one point, there is an infinity of bidding curves solution. These curves only need to intersect the bidding curve S of the rival on the chosen point.

Assuming that if the company plays S against S, each company will be given $\frac{1}{2}cU$. The profit will be $\Pi_S(S) = \frac{1}{8}cU^2$. Now, if the company plays $M(p)$ against S, then its output would be $r = \frac{2U}{3} + \frac{p}{3c}$. Thus, its profit would be $\Pi_{M(p)}(S) = \frac{1}{2}cr^2 - pr = \frac{1}{18c}(2cU + p)(2cU - 5p)$. The following graph shows the profits.

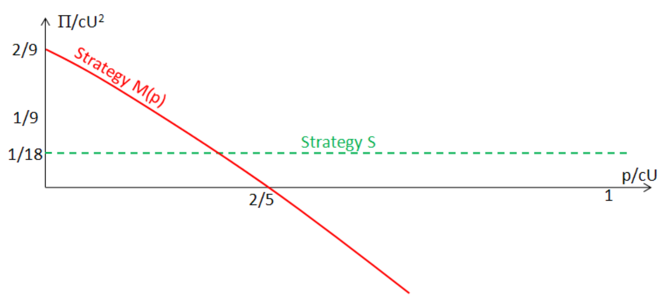

Figure 6.9: Profits of different strategies against S

Note that the profit is negative when a company sets a high line price against strategy S.

6.3.3 Strategies $S(p)$

Now, we will introduce the strategies $S(p)$ which bidding curves are S for prices higher than p, and equal to p for higher demands. $S(p + \epsilon)$ is one of the best reply to $M(p)$, and is a best reply to $S(p)$ when p is low enough. Therefore, we will now suppose that companies only play curves $S(p)$. The game is illustrated by figure 6.10.

Their profits can be read on figure 6.11.

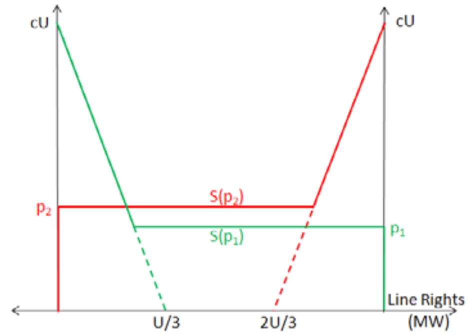

Figure 6.10: Profits $\Pi_{S(p_1)}(S(p_2))$ of strategies $S(p_1)$ against $S(p_2)$

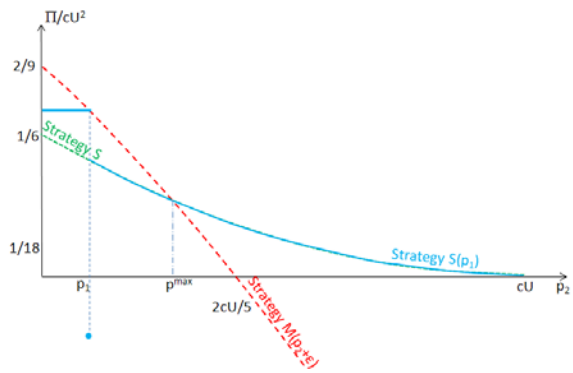

Figure 6.11: Profits $\Pi_{S(p_1)}(S(p_2))$ of strategies $S(p_1)$ against $S(p_2)$

Notice that playing $S(p)$ with $p \geq p^{max}$ is always less profitable than playing any $S(p)$ with $p \leq p^{max}$. Moreover, note that the best reply to $S(p)$ with $p < p^{max}$ is $S(p+\epsilon)$. However, $S(p^{max})$ is a bad reply to $S(p^{max})$ because companies would have to share the line at high cost.

6.3.4 Nash Equilibria

We now have the two following results corresponding to the intersection of the red dashed curve and the blue curve of figure 6.11.

Theorem 9. *If the prices are continuous, then the situation where both companies choose randomly a strategy $S(p)$ with $p \in [p^{max} - \epsilon, p^{max}]$ are ϵ-equilibrium in mixed strategies.*

Theorem 10. *If the prices are not continuous, then we have the highest price less than p^{max}. Call it $p^{max} - \epsilon$. Then the strategies $(S(p^{max}), S(p^{max}) - \epsilon)$ and $(S(p^{max}) - \epsilon, S(p^{max}))$ are Nash Equilibria.*

In both cases, despite we have first constrained the strategies to be $S(p)$ strategies, these are actual ϵ-equilibrium and Nash Equilibrium. As a matter of fact, given the curve $S(p)$ of its rival, the set of the best replies of a company are the set of decreasing functions that contains one of the points $(q_1, p_1) \in \mathcal{S}$, where $\mathcal{S}$ is the set of points on the rival's bidding curve that maximize the profit of our company. But if the rival plays $S(p)$, then $\mathcal{S}$ is (approximatively in the continuous case) the actual set of possible dispatches of our equilibria.

6.3.5 Mixture

Let us now look for Nash Equilibria in mixed strategies. In order to do that, we will consider the distribution $f_k : [0, p^{max}] \rightarrow \mathbb{R}_+$ of values of p corresponding to strategies $S(p)$ for company $k \in \{1, 2\}$. We have the constraint $\forall k, \int_0^{cU} f_k = 1$.

Now, each company wants to maximize its profit. For company 1, we have

$$\Pi_1(f_1, f_2) = \int_0^{p^{max}} \Pi_{S(p_1)}(S(p_2)) f_1(p_1) f_2(p_2) dp_1 dp_2$$

Now, a Nash equilibrium in mixed strategy would be a value of (f_1, f_2) so that $f_1 \in \text{Argmax}\, \Pi_1(f_1, f_2)$ and $f_2 \in \text{Argmax}\, \Pi_2(f_1, f_2)$. Thus, we need to have

$$\text{Supp}\, f_1 \subset \text{Argmax}_{p_1} \int_0^{p^{max}} \Pi_{S(p_1)}(S(p_2)) f_2(p_2) dp_2$$

Since $\Pi_{S(p_1)}(S(p_2))$ is a piecewise polynomial of the second degree, the condition of maximality leads after derivation and integration by parts to a differential equation of the second order, with two primitives of f_2. As the equations are quite complicated, we will not write them here.

I have not been able to determine the existence of such equilibria. However, we can say that if there is such an equilibrium, then the expected incomes of the companies will be higher than in the previously mentionned equilibrium in mixed strategy, as profits are always higher, as displayed on figure 6.9.

6.3.6 Conclusion

The following theorem sums up the study of this section.

Theorem 11. *Nash equilibria in pure strategy are blue points. Equilibria in mixed strategy restricted to $M(p)$ and $S(p)$ strategies are necessarily mixture of $S(p)$ strategies, that lead to distributions of the yellow lines and the blue points.*

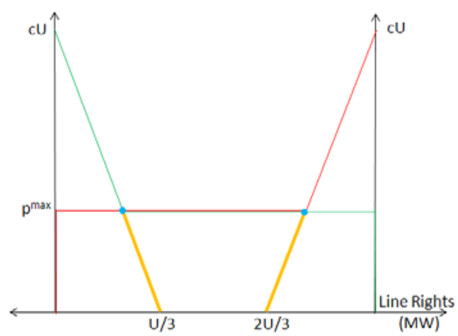

Figure 6.12: Equilibria

Chapter 7

Game on line rights with uncertain demand

We still consider the pro-rata-with-incremental-output allocation. This time, demand is random with a known probability distribution.

7.1 Generalities about supply function equilibria with uncertain demand

In this section, we are going to use the principles of Supply Function Equilibrium introduced in [3], and the methods presented in [1], in order to study the existence of Nash Equilibria.

7.1.1 Assumptions and notations

We are now supposing that we have a duopoly, where companies 1 and 2 have to announce positive decreasing bidding curves. Depending on the value of the demand h and on these curves, the System Operator then choose a clearing line rights price p^*, so that the outputs associated to this line rights price in the bidding curves may fulfill the demand.

Note that the demand is uncertain. However, it has a cumulative distribution function F, known by all the agents of the market.

Suppose company 2 has a bidding curve $s(p)$. We are now going to look for the best reply to such a curve. In order to do that, we will consider a parametrized curve $(q(t), p(t))$ for $t \in [0, T]$.

Now, suppose that the dispatch of company 1 is $[q, q+dq] \times [p, p+dp]$. That means, that the clearing price is in $[p, p+\mathrm{d}p]$, which means that company 2's dispatch is in $[s(p), s(p)+s'(p)dp]$ with an approximation at first order. Thus, demand is in $[q+s(p), q+s(p)+dq+s'(p)dp] = [h, h+dh]$. Hence, the deviation of demand corresponding to a deviation (dq, dp) is $dh = dq + s'(p)dp$.

Let $\psi(q,p) = \Pr(d < q + s(p)) = F(q + s(p))$. Then the probability that $[q, q+dq] \times [p, p+dp]$ is dispatched is

$$\begin{aligned} d\psi &= F(q + s(p) + dh) - F(q + s(p)) \\ &= F'(q + s(p))dh \\ &= (dq + s'(p)dp)F'(q + s(p)) \end{aligned}$$

If (q, p) is dispatched, then the profits of company 1 regardless to line rights costs is $\Pi(q, p)$. Note that in plenty of cases this profit does not depend on p, since p is just a clearing price of the line rights and does not interfere with the main market of company 1. Yet, in this case, it actually affects the output of company 2, which affects the electricity price, hence affects the profits of company 1.

If we now consider the line rights costs, the global profit of company 1 when (q, p) is dispatched is $R(q, p) = \Pi(q, p) - pq$. The global expected profit can now be calculated given the demand distribution and bidding curve $\Gamma = \{(q(t), p(t)), t \in [0, T]\}$:

$$\mathbb{E}[R] = \int_{\Gamma} R(q, p) d\psi(q, p)$$

7.1.2 The function Z

In order to maximize the expected profit, Anderson and Philpott have introduced in [1] the following function :

$$Z(q,p) = R_q\psi_p - R_p\psi_q$$

where the partial derivative of a function f by some variable x is noted f_x. Thanks to Green's Theorem in the plane, we have the following theorem.

Theorem 12. *Let γ be a loop curve, and $\mathcal{S}$ the delimited area, counted positive if γ goes in an anticlockwise direction. Then,*

$$\iint_{\mathcal{S}} Z(q,p)dqdp = \int_\gamma R(q,p)d\psi(q,p)$$

Proof. Let $G = R\psi_p$ and $H = R\psi_q$. Then

$$\begin{aligned} G_q - H_p &= R\frac{\partial^2\psi}{\partial q\partial p} + R_q\psi_p - R\frac{\partial^2\psi}{\partial q\partial p} - R_{p\psi q} \\ &= R_q\psi_p - R_p\psi_q = Z \end{aligned}$$

Therefore, applying Green's Theorem,

$$\begin{aligned} \iint_{\mathcal{S}} Zdpdq &= \iint_{\mathcal{S}} (G_q - H_p)dqdp = \int_\gamma Hdq + Gdp \\ &= \int_\gamma R\psi_q dq + R\psi_p dp = \int_\gamma Rd\psi \end{aligned}$$

Hence the theorem. □

7.1.3 Utilisation of Z through an example

Thanks to this theorem, we now have a method with other theorems to use. However, because those theorems are quite complicated to formulate, we will just show them through an example.

Consider the following figure.

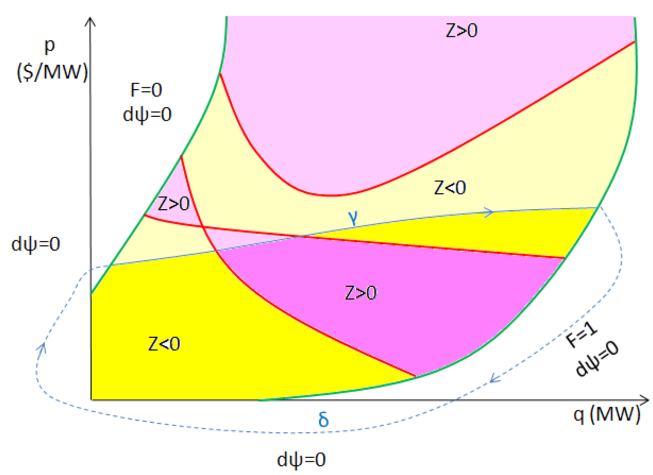

Figure 7.1: Example of a Z function

The yellow areas are where $Z < 0$, the pink areas where $Z > 0$ and the red curves are where $Z = 0$ but $d\psi \neq 0$. Now, the company has to choose a bidding curve γ. This curve necessarily goes from the area $F = 0$ to the area $F = 1$. Its value inside those areas will not affect its expected profit, as there is no distribution of demand for him there.

Notice that we can complete the curve γ with some curve like δ, which satisfies $d\psi_{|\delta} = 0$, so that $\gamma \cup \delta$ is the contour of an area $\mathcal{S}$. We can now apply the previous theorem. Thus,

$$\int_\gamma R d\psi = \int_\gamma R d\psi + \underbrace{\int_\delta R d\psi}_{\text{nilas } d\psi_{|\delta}=0} = \int_{\gamma\cup\delta} R d\psi = -\iint_{\mathcal{S}} Z$$

This value is represented by the darker areas in the figure.

Note the negative sign due to the anticlockwise direction of $\gamma \cup \delta$. Therefore, the problem is to find a curve under which the yellow areas minus the pink areas will be the highest.

Given like that, the problem is not well written. As a matter of fact a curve with anticlockwise loops in the yellow area or clockwise loops in the pink area would have an expected value associated to this curve as high as we want.

7.1.4 Shape of solutions with the curve being a function constraint

Therefore, we will add constraints. The curve we are looking for must be continuously parametrized $(q(t), p(t))$ for $t \in [0, T]$, with $q(t)$ increasing. This means that it has to go from the left to the right. Moreover, if $q(t)$ is stationnary in an interval, then $p(t)$ must be monotone in this interval too.

Theorem 13. *In this example and with these constraints, the solution is necessarily a sequence of those two kinds of curve :*

- *The bright blue curves : defined by $Z(q, p) = 0$, with a pink area right above, and a yellow one just below.*
- *The dark blue lines : defined as vertical lines crossing pink and yellow areas, for which the integral of Z along it is nil. The area on top must be yellow, the one in the bottom must be pink.*

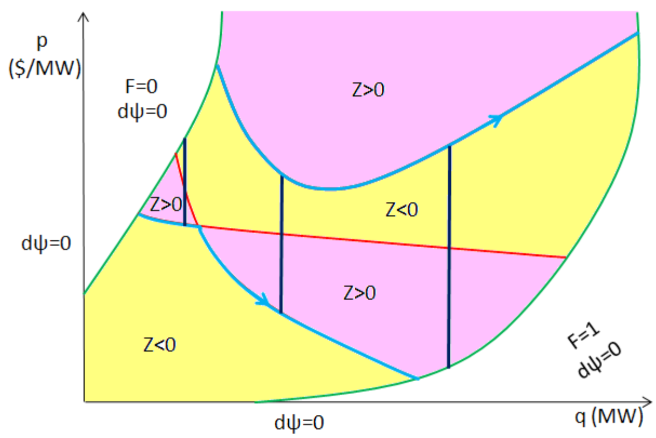

Figure 7.2: Solutions of the bidding curve problem

Proof. Suppose γ is an admissible curve with a point where $Z < 0$. By continuity, there is a portion of γ around this point where $Z < 0$.

Suppose also that the curve is not vertical in this area. Then, consider a loop δ that coincides with γ along this portion but goes in the opposite direction, and then turns back in a clockwise direction while remaining in the yellow area, as shown in figure 7.3.

In this figure, γ is blue and orange, whereas δ is green and orange. Now, $\gamma\cup\delta$ is an admissible curve, as their common portion delete themselves. Yet, since δ is a clockwise loop in the yellow area, $\int_\delta R d\psi > 0$. Hence, $\int_{\gamma\cup\delta} R d\psi > \int_\gamma R d\psi$, which shows that γ is not an optimal solution.

A similar proof with an anticlockwise loop would show that an optimal solution cannot have a point where $Z > 0$. Thus, a solution is necessarily included in $Z^{-1}(0)$.

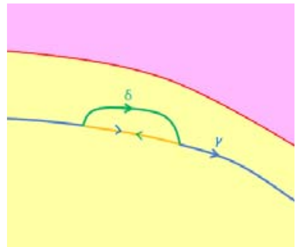

Figure 7.3: Non-optimality of γ

Now, if a yellow area is right above a curve (resp. a pink area right below), then we can still use the same reasoning to show that this curve cannot be optimal.

We have almost proved the result. Yet, the optimal curve may cross pink and yellow areas, but this would be with a vertical line. Moreover, the upper limit has to be a point of a light blue curve, or of a green one, since, otherwise the previous section of the curve could be dealt with as we previously did with non-vertical curves. It is the same for the lower limit.

Yet, note that if we move the vertical curve whose absciss is q to an absciss $q + dq$. The intersections (q_1, p_1) and (q_2, p_2) (with $p_1 < p_2$) of this line with the light blue curves will also move. In our example, this displacement is continuous. Therefore, if the vertical line is run from up to down, then the variation of the integral due to this displacement will be $-dq \int_{p_1}^{p_2} Z(q,p)dp$ at first order. If the line is run from down to up, then it is the opposite value.

The condition of optimality now leads to $\int_{p_1}^{p_2} Z(q,p)dp = 0$, which is the last thing we needed to prove our result. □

Then, depending on the sign of the integral of Z in the areas delimited by those blue lines (and the green ones), we can determine which combination of blue lines will maximize the integral of Z under the curve, which is the solution of our expected profit maximizing problem.

7.1.5 Actual solution with the curve being a function constraint

In order to find the right combination of those curves, we should have a look at the sign of the integrals of Z in areas delimited by blue and green lines. This is displayed by the following figure, where the positive areas are painted in pink, and the negative ones in yellow.

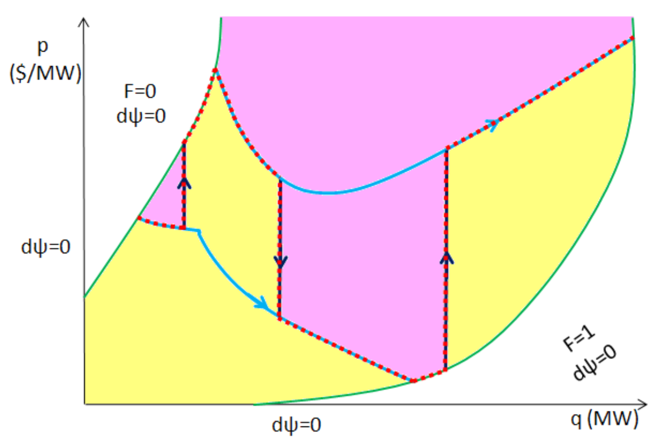

Figure 7.4: Solution of the bidding curve problem

Recall that the integral of Z over the whole area is the opposite of the expected profit. Therefore, we want to encompass as much yellow area as possible. Hence, the solution is the dashed red curve.

Note that the vertical lines can now be oriented, depending on the colour of the area on their left..

7.1.6 Bidding curve problem

Yet, in order to organize an auction, the System Operator may force the companies to bid decreasing curve. It means that we are still looking for a parametrized curve with $q(t)$ increasing, but, now, $p(t)$ has to be decreasing.

Although the lower blue curve remains admissible, the higher one is not. Yet, it is possible to modify it slightly to get it admissible.

The use of Z gets more complicated, as the next subsection will show, despite the following result.

Theorem 14. *The solution is necessary a succession of decreasing light blue curve, vertical and horizontal line. Horizontal (resp. vertical) lines must begin in a pink (resp. yellow) area or a light blue line, and it must end in yellow (resp. pink) areas or a light blue line. The vertical and horizontal sections necessary satisfy* $\int_{\text{line}} Z = 0$.

Proof. The same reasoning as previously applies to curves not horizontal nor vertical. Hence the first sentence of the theorem.

If a horizontal or a vertical section links two blue or green lines, then we can use the same reasoning as previously to prove the necessity that $\int_{\text{line}} Z = 0$.

If it does not, then it means we have a succession of a horizontal and a vertical curve. Suppose we have first the horizontal and then the vertical curve. Consider (q, p) the point where the two lines meet. If this point is in pink area, then the expected revenue can be improved by the following dark dashed line.

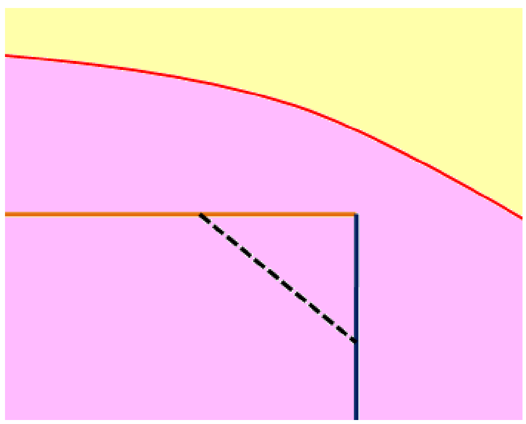

Figure 7.5: Reason why (q, p) needs to be in a yellow area

A similar explanation justifies that a succession of a vertical then a horizontal line needs to cannot occur in a yellow area. We have just proved the second sentence of the theorem.

By continuity of Z, as the horizontal line starts in a yellow area from a vertical line or on a light blue line, we can move slightly the horizontal line up or down with it still satisfying the two first conditions, and add or withdraw a little part of the following vertical line. In fact, by doing that, we obtain a new admissible curve. By displacing the horizontal section from p to $p + dp$, we add to the expected profit the value $dp \int_{horizontal\ line} Z$. We can do the same reasoning for the vertical part.

In fact, as we consider the set of curves with slight displacements of both the vertical and the horizontal sections, we can write the expected profit as a function of the meeting point (q, p). If our curve is solution, it means that it maximizes the expected profit among all the decreasing positive bids, hence, in particular, among all those curves.

Necessarily, the differential of the expected profit by (q, p) needs to be nil, which means for both the vertical and the horizontal lines $\int_{line} Z = 0$. Hence the third and last sentence of our theorem. □

Despite this theorem, solutions remain difficult to characterize. In our example, let us introduce the horizontal orange line drawn on figure 7.6, and for which we have $\int_{\text{line}} Z = 0$. Supposing that this is the only horizontal line that satisfies the condition of the previous theorem, the optimal bidding curve is a composition of blue and orange lines.

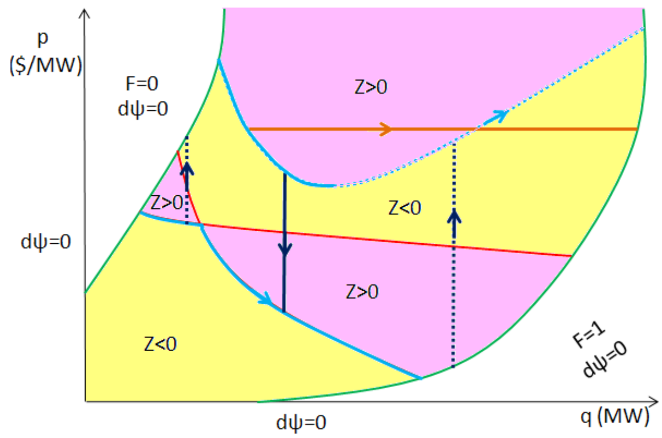

Figure 7.6: Components of the solution of the bidding curve problem

There are three local maxima now that we have to compare to find the global maximum of our problem. These three curves are represented in the figure 7.7. The first curve is the light blue then dark blue curve. The second is the green then purple then dark blue curve. The third is the green then orange curve.

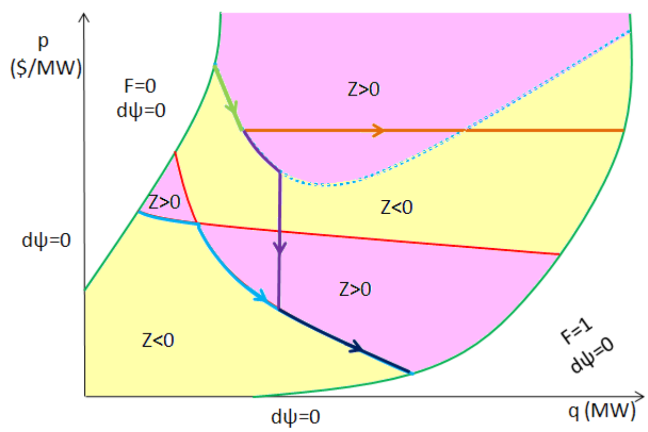

Figure 7.7: Solutions of the bidding curve problem

7.1.7 Difficulties of the utilisation of Z

Let us look at a tricky case, where horizontal and vertical sections meet in a yellow area. We may assume here that in the yellow area $Z = -1$, whereas in the pink area $Z = 1$. The case is represented on figure 7.8.

The blue (resp. green) dashed line is the the set of points from which a vertical (resp. horizontal) line that satisfies $\int_{\text{curve}} Z = 0$ can be drawn.

Their intersection leads to a point, for which both vertical and horizontal lines satisfy the constraint. Since it is the only one with that property, the orange then purple curve is necessarily the optimum.

7.1.8 Example

In this part, we will show simple examples, where the usage of Z enables a simple resolution. Supp F' is assumed to be $[0, 1]$.

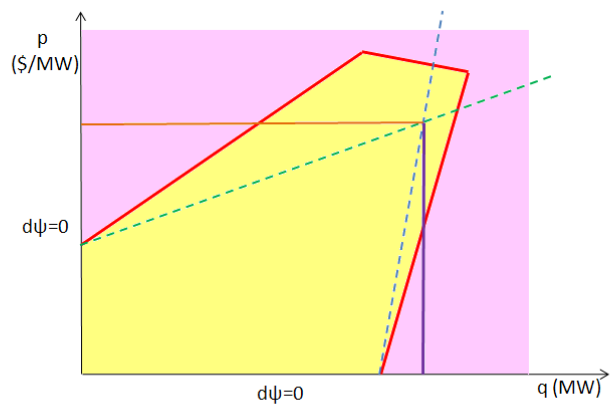

Figure 7.8: Tricky case

Linear profit function

If $s(q) = a - q$ and $\Pi(q) = q$, then the optimal reponse is the curve defined by

$$p + q - 1 = 0$$

which gives a decreasing linear curve, positive for $(p, q) \in \operatorname{Supp} d\psi$.

If $a = 1$, then we have a symmetric Nash Equilibrium.

Quadratic profit function

Suppose $s(q) = a - bq$ and $\Pi(q) = kq - q^2$. Then

$$\begin{aligned} Z(q,p) &= bp + q - k + 2q \\ &= 0 \end{aligned}$$

Then the optimal reponse is the curve defined by

$$p = k/b - 3q/b$$

If $b = \sqrt{3}$ and $a = k/\sqrt{3}$, then we have a symmetric Nash Equilibrium. Note that if k is not high enough then these bids can be negative, which would not correspond to auctions. In fact when $k \leq 3$, this method does not lead to admissible curves. Yet, if $3/2 \leq k \leq 3$, then curves defined by

$$p = \begin{cases} k/b - 3q/b, & q < 1/2 \\ 0, & q \geq 1/2 \end{cases}$$

are Nash Equilibria.

7.2 Application to the line rights market with high fixed electricity price

We are now going to apply this Z function to the study of the HVDC line rights market, in a simplified case where southern electricity spot prices are supposed to be fixed and high.

7.2.1 The Z function

In order to keep it simple, we will assume that the price π is given by northern generators and that southern generators cannot affect it. Moreover, marginal costs will be linear, equal to q.

Then the profit can be written $\Pi(q,p) = \pi q - q^2/2$. Then, $R(q,p) = \pi q - q^2/2 - pq$. Hence,

$$Z(q,p) = R_q\psi_p - R_p\psi_q = [(\pi - q - p)s'(p) + q]F'(q + s(p)).$$

When $F'(q + s(p)) \neq 0$, we have

$$Z = 0 \Leftrightarrow q = \frac{-s'(p)(\pi - p)}{1 - s'(p)}$$

Note that if s is convex and decreasing, then $-s'$ is a positive decreasing function. Hence, $\dfrac{-s'(p)}{1 - s'(p)}$ is positive decreasing too, as well as $\pi - p$ if $p < \pi$ (which is necessarily true when profit is non-negative). Therefore, $Z = 0$ corresponds to a positive decreasing curve.

7.2.2 Symmetric Nash Equilibrium

Let us now look for a symmetric Nash Equilibrium. In this case, necessarily, $s(p) = q(p)$, which implies that the curve $q(p)$ must be a solution of the differential equation $y' = -\dfrac{y}{\pi - y - x}$. The inverse equation for $x(y)$ is

$$\begin{aligned}
x' &= -\frac{\pi - x - y}{y} \\
x' - \frac{x}{y} &= -\frac{\pi - y}{y} \\
\frac{x'}{y} - \frac{x}{y^2} &= -\frac{\pi - y}{y^2} \\
\left(\frac{x}{y}\right)' &= -\frac{\pi}{y^2} + \frac{1}{y} \\
\frac{x}{y} &= \frac{\pi}{y} + \log y + \alpha \\
x &= \pi + y \log y + \alpha y
\end{aligned}$$

Note that $\tilde{p}_\alpha(q) = \pi + q \log q + \alpha q$ is a decreasing curve, as long as $q \leq e^{-1-\alpha}$. Suppose Supp $F' =]0, 1[$.

Theorem 15. *For $\alpha \in [-2\pi + \log 2, -1 + \log 2]$, bidding curves equal to p_α in $[0, 1/2]$ and horizontal equal to $p_\alpha(1/2)$ in $[1/2, 1]$ form symmetric Nash Equilibria.*

Proof. Note that those curves are decreasing positive functions. Hence they are admissible.

If the opponent plays such a curve, then, as Z is a function defined locally, the curve $Z = 0$ for $q < 1/2$ is $\tilde{p}_\alpha$. Under it and above the line $p = p_\alpha(1/2)$, we have $Z < 0$. Under the line, $Z = 0$. Above p_α, $Z > 0$. When $q > 1/2$, we have $Z = 0$ under the curve $1 - p_\alpha$, and $Z > 0$ above it. The optimal response needs to be a curve p_α in $[0, 1/2]$, but there is no constraint except being decreasing and positive in $[1/2, 1]$.

Hence the curve of this opponent is one of the best response, which means that we have a Nash Equilibrium. □

7.2.3 Profits

Because we are looking for symmetric Nash Equilibria, the companies will always dispatch half of the total demand. Hence, their profits are equal and are

$$\mathbb{E}[R] = \int_0^1 \left(\pi\frac{h}{2} - \frac{h^2}{8} - \tilde{p}_\alpha\left(\frac{h}{2}\right)\frac{h}{2}\right)F'(h)dh.$$

As $\tilde{p}_\alpha$ is obviously an increasing function of α, the expected is a decreasing function of it. In fact, the expected profit can be written $\mathbb{E}[R] = R_0 - \alpha\dfrac{\mathbb{E}[h^2]}{4}$, where h is the total demand random variable and where R_0 is a constant independent from α.

Note that the best Nash Equilibrium for companies requires α to be as low as possible, hence $\alpha = -2\pi + \log 2$. This condition also means that the bid for the purchase of half of line rights is 0.

7.2.4 Asymmetric marginal costs

Let us now consider an asymmetric case, where marginal costs are $c_1(q) = c_1 q$ and $c_2(q) = c_2 q$. If bids at equilibrium are s_1 and s_2, then we have the following differential equation.

$$\begin{cases} s_1' = -\dfrac{s_2}{\pi - p - c_2 s_2} \\ s_2' = -\dfrac{s_1}{\pi - p - c_1 s_1} \end{cases}$$

In order to solve this system we follow the approach of Anderson and Hu [?]. We divide $[0,1]$ into n equally long sections, and we define the $4(n+1)$ values of s_1, s_1', s_2 and s_2' at each extremity of the sections. Add the relations corresponding to the differential equations. Also add the constraints that, for each section, tangents of each extremity must meet between the extrimity. Add $s_1(0) = s_2(0) = \pi$.

We now have nonlinear programming problem that can be solved by GAMS/CONOPT (the source code is provided in Appendix 1). The solution is plotted in fugure 7.9.

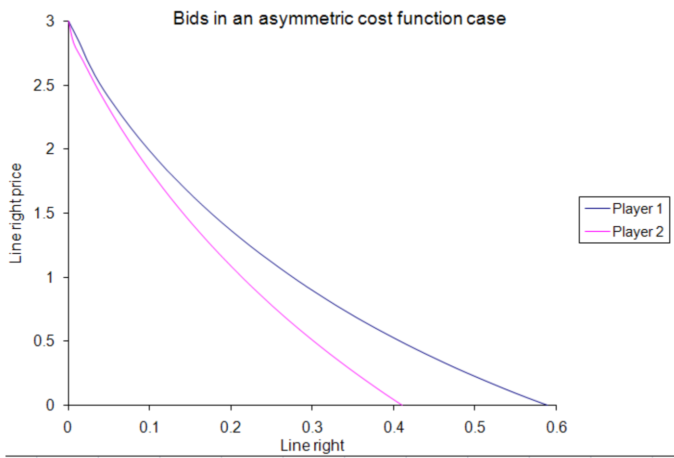

Figure 7.9: Nash Equilibrium in an asymmetric case

Here we have $\pi = 3$, $c_1 = 1$ and $c_2 = 5$. The same reasoning as theorem 15 proves that these curves form a Nash Equilibrium as long as the first bids at line price 0 have a sum equal to 1.

7.2.5 Efficiency

It is interesting to look at the efficiency of such a dispatch. An efficient dispatch needs to satisfy $c_1 q_1 = c_2 q_2$, which means that $q_1/q_2 = c_2/c_1 = 5$. However, the value of q_1/q_2 remains close to 1,3, which means that the dispatch is not efficient.

In an efficient dispatch, when demand is h, the dispatches are $q_1 = 5h/6$ and $q_2 = h/6$. And the global cost is $c_1 q_1 + c_2 q_2 = 10h/6$. Here, however, as $q_1 \approx 1.3 q_2$, we have $q_1 = 1.3h/2.3$ and $q_2 = h/2.3$. Hence the global cost is $c_1 q_1 + c_2 q_2 = (1.3+5)h/2.3 = 6.3h/2.3$. The global efficiency has been multiplied by $(6.3/2.3)/(10/6) \approx 1.6$. Numerically, we have found 1.625.

The effect of the auction is to bring closer the dispatches of the two companies.

7.2.6 Oligoply

Suppose that there are more companies $k \in K$ with marginal costs $(c_k)_{k \in K}$. Then the differential equation for the bids $(s_k)_{k \in K}$ can be written

$$\forall k, \sum_{j \neq k} s_j' = -\frac{s_k}{\pi - p - c_k s_k}.$$

This equation can be written $As = F(s)$, where $A = 1_K - I_K$, with 1_K the matrix equal to 1 everywhere and I_K the identity matrix, and $(F(s))_k = -\frac{s_k}{\pi - p - c_k s_k}$. But A is invertible, and $A^{-1} = \frac{1}{n-1}(1_K - (n-1)I_K)$. Hence, $s' = A^{-1}F(s)$.

A similar computation as above can be used to solve this equation.

7.3 Real electricity price fixing system

The previous part dealt with a simplified model where electricity spot price was constant and independent of the line rights auction. However, this is not how things work. Although the problem is difficult, we are now going to study what happens where the price actually depends on the dispatches.

7.3.1 Equations

Suppose that $\Pi(q)$ is determined by the real price fixing system. Suppose player 1 has cost $C_1(q)$ and player 2 has cost $C_2(q)$. Suppose also that Player 2 bids the positive decreasing curve $s(p)$. Thus if Player 1 bids (q,p) and $C_1'(q) < C_2'(s(p))$, then Player 2 sets the energy price. Otherwise Player 1 sets the energy price. Thus,

$$\Pi(q,p) = \begin{cases} qC_2'(s(p)) - C_1(q), & C'(q) < C'(s(p)) \\ qC_1'(q) - C_1(q), & C'(q) \geq C'(s(p)) \end{cases}$$

Suppose $C_1(q) = C_2(q) = \frac{q^2}{2}$. Then

$$\Pi(q,p) = \begin{cases} qs(p) - q^2/2, & q < s(p^-) \\ q^2/2, & q > s(p^+) \end{cases}$$

Notice that there is a case with horizontal sections that we did not write down.

The Z function can now be evaluated as follows.

$$\begin{aligned} Z &= R_q\psi_p - R_p\psi_q = \big((\Pi_q - p)\,s' - (\Pi_p - q)\big)F' \\ &= \begin{cases} Z_1 = [(-s')(p + 2q - s) + q]F', & q < s(p) \\ Z_2 = [(-s')(p - q) + q]F', & q > s(p) \end{cases} \end{aligned}$$

Now, we can notice these important functions q of p.

$$Z_1 = 0 \text{ and } (q,p) \notin \text{Supp } d\psi \quad \Leftrightarrow \quad q = \frac{(-s')(s-p)}{1 + 2 \times (-s')}$$

$$Z_2 = 0 \text{ and } (q,p) \notin \text{Supp } d\psi \quad \Leftrightarrow \quad q = \frac{(-s')p}{(-s') - 1}$$

7.3.2 Values of the Z function

Notice that s is decreasing, thus $s - p$ is decreasing too. Suppose that $-s'$ is not too much increasing (s not too much concave), then $\frac{(-s')(s-p)}{1 + 2 \times (-s')}$ may be decreasing.

If $(-s') \leq 1$, then the curve corresponding to Z_2 is not admissible. Otherwise, $\frac{(-s')}{(-s') - 1}$ is higher than 1, and, supposing that s convex, we have $(-s')$ decreasing. Hence, we may assume that when p gets low, $\frac{(-s')}{(-s') - 1}$ is close to 1. This would mean that curve $q = p$ is an asymptote to $q = \frac{(-s')p}{(-s') - 1}$.

The yellow area is a part where $F'(q,p)$ is nil. The pink area is where the second players sets the price. The orange area corresponds to a pricing by the first player. Darker areas correspond to $Z < 0$. The two red curves are $Z = 0$ lines.

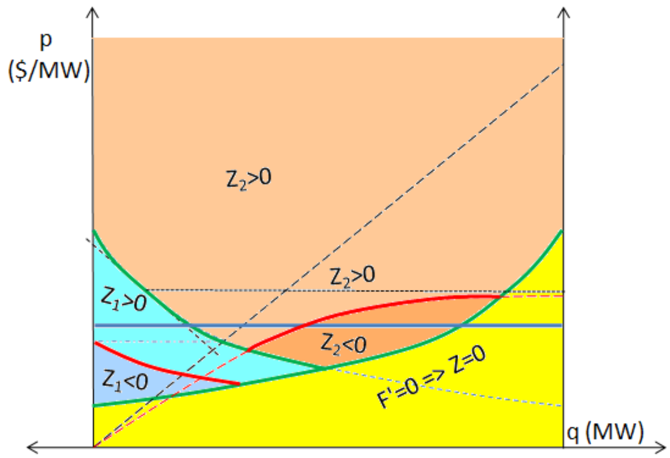

Figure 7.10: Plot of $Z(p, q)$

7.3.3 Strategies

There are two kinds of strategies. On the one hand, the company can try to take advantage of the high price set by the other company. If it does that, it would bid the lower red curve. Then, in the yellow area, it can bid any decreasing curve. It won't matter since this part of the curve won't be used for the choice of the allocation.

On the other hand the company can try to set the price itself, and bid accordingly to the higher red curve. Yet, because of the decreasing constraint, its optimal solution will be a horizontal linear curve, like the blue one. As a matter of fact, when $Z < 0$, we would want the curve to be lower, but it cannot be lower than its value in the area $Z > 0$, because it comes before. Similarly, when $Z > 0$, we would want the curve to be higher but it cannot be.

Among all the horizontal curves, the best one will be the one that leads to $\int_{curve} Z = 0$. If none of them manage to do that, then this would mean that they all have positive values. Hence, they are all worse than bidding nothing.

7.3.4 Symmetric Nash Equilibria

Now, let us look for Nash Equilibria in pure strategies. In a symmetric Nash equilibrium, both players would bid the same decreasing bidding curve $s(p)$. Thus, we would have either $Z_1(s(p), p) = 0$ or $Z_2(s(p), p) = 0$. In both cases, these equations lead to differential equations for s :

$$s' = \frac{s}{s \pm p}$$

As we inverte s to obtain $p(q)$, we have the equation $p' - \frac{p}{q} = \pm 1$, which, for $q > 0$, is equivalent to $(\frac{p}{q})' = \pm\frac{1}{q}$. Then, we have $p(q) = \pm q \log q + \alpha q$.

Notice that if we consider $Z_1 = 0$, then it is a positive sign, which means that the bid is necessarily negative for low value of q, and increasing later, which is not acceptable. Yet, if the sign is negative, then the bid is necessarily increasing at first.

Anyway, it seems weird to consider two players whose strategies are the same. As a matter of fact, we have just studied cases where they both want to be the one who set the electricity price, or they both want the other to set the price. In the end, because of the symmetry, they just share the line. Although the dispatch is efficient, the two companies actually set the price very low, which is not in their best interest.

7.3.5 Asymmetric Nash Equilibria

Therefore, we may look into the case where company 2 tries to set a high electricity price, whereas company 1 tries to take advantage of it. This means that they have different bidding curves $s_1 \leq s_2$, and we have the following differential equations :

$$\begin{cases} s_1' = \dfrac{s_2}{2s_2 - s_1 + p} \\ s_2' = \dfrac{s_1}{p - s_1} \end{cases}$$

But, once again, for (s_1, s_2) to be a solution of our problem, we need to have $0 < s_1 \leq s_2$, and this implies $s_1' > 0$. Thus, s_1 is increasing and is not a bid function.

In fact, figure 7.10 shows that there cannot be any Nash Equilibrium pair of curves without a horizontal section. As a matter of fact, the best response to a decreasing curve is necessarily either a strictly lesser one (the lower red curve) or a horizontal curve.

7.3.6 Mixtures

But then, we have an overcutting problem among horizontal curves, which has no Nash Equilibrium in pure strategies. The solution of such a problem becomes a solution in mixed strategies, where the companies will alternatively take a large possession of the line rights. Efficiency will not be reached.

The problem in mixed strategy here is very difficult. When price is low, the best response to a horizontal line is a horizontal line a little bit higher. But when price is high, it is a piecewise curve, first a little bit higher until a certain output after which the curve's only constraint is to be lower. As opposed to the study we did with no uncertainty on demand, there is no curve that is a best response to all the horizontal bids.

Chapter 8

A pro rata with incremental benefit allocation

As we have seen, the current allocation and the pro rata with incremental output are not satisfactory. Therefore, we will now propose another allocation system.

8.1 Description of the allocation

The idea of this system is to guarantee efficiency and fairness. In order to do that, we will simply impose the dispatch, so that no game on line rights can happen, which will enable better efficiency. Moreover, our idea of fairness is that companies who take advantage of the line should be the ones who pay for it. Yet, a company's goal is to maximize its profit. That's why we have come up with a tax pro rata with incremental benefits due to the HVDC line. These incremental benefits are the difference between the benefits with a line, and the benefits had the line been cut.

For every trading period of 30 minutes, an expected demand is estimated, and companies give announced marginal cost curves (or step functions). Given that, the System Operator calculates the dispatch with the HVDC line, and the dispatch with the HVDC line cut. He calculates the profits of each company, supposing that they have announced their real marginal costs. This leads to estimated incremental benefits. Then, each company has the dispatch calculated with the HVDC line, and pays a tax pro rata with the estimated incremental benefits.

Therefore, the game for each company is now only on the choice of the costs announced for a year. This is the game we are now going to study.

8.2 Game problem

The case where the System Operator actually knows the real profits leads to a more simple problem. That is why this is what we will start with.

8.2.1 Supposing that the System Operator knows the real profits

Now, consider a company. It has to choose an electricity bidding curve via the announced marginal cost $c(q)$ that maximizes its expected profit, given its real marginal cost $\bar{c}(q)$, the aggregated bid of the other companies $\check{c}(q)$ and a distribution of the demand f. A solution of this optimization problem is a best reply to the other companies' bids.

Given a demand and the aggregated bidding curve of the other companies, our company needs to choose an output q^0, that corresponds to a point on the curve $\check{c}(d-q)$. Let $\Pi^0(q^0)$ be the profit related to this output. Similarly, we define $\Pi^1(q^1)$ the profit related to an output q^1 on the curve $\check{c}(d+U-q)$.

Then, if α is the percentage of tax on incremental profits, the problem for our company is now :

$$\max_{q^0,q^1} \Pi^0(q^0) + (1-\alpha)(\Pi^1(q^1) - \Pi^0(q^0))$$

$$\text{Subject to } : q^0 \leq q^1 \leq q^0 + U$$

This problem can also be written as follows :

$$\max_{q^0,q^1} (1-\alpha)\Pi^1(q^1) + \alpha\Pi^0(q^0)$$

$$\text{Subject to } : q^0 \le q^1 \le q^0 + U$$

Yet, the problem of maximizing $\Pi^0(q_0)$ (resp. $\Pi^1(q_1)$) with no constraint on q_0 (resp. q_1) is the one we discussed in section 3. Suppose we have solved the problem of section 3 and found a solution $c^*(q)$. Then, as shown on the following figure, the intersections q^0 and q^1 of this curve with $\breve{c}(d-q)$ and $\breve{c}(d+U-q)$ obviously satisfy the constraints.

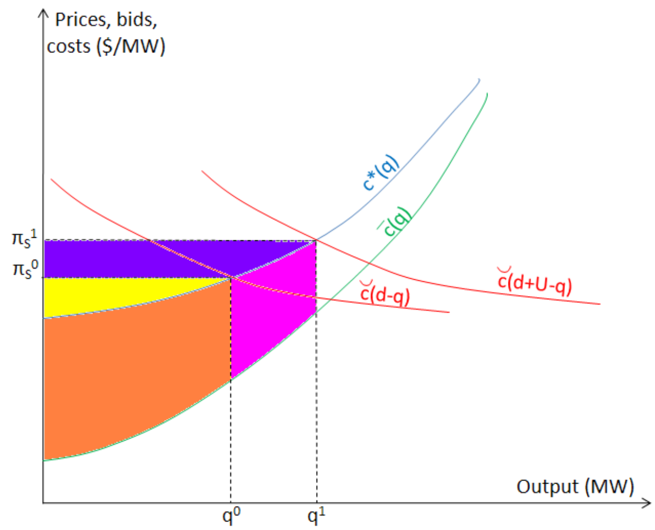

Figure 8.1: Bids in a pro rata with incremental profits allocation

Therefore, $(1-\alpha)\Pi^1(q^1)+\alpha\Pi^0(q^0)$ is a reachable upper bound to our objective function, hence (q^0, q^1) is the solution. And $c^*(q)$ is the optimal bidding curve for the company. Hence the following theorem.

Theorem 16. *Nash Equilibria of the problem discussed in 3.2. are Nash Equilibria of this problem too.*

8.2.2 The System Operator estimates incremental profits

In this model the System Operator estimates profits from the announced costs. The difference is that the bright pink area, located between q^0 and q^1 and delimited by the curves $\bar{c}$ and c is not taxed. The maximization problem has become :

$$\max_{q^0,q^1} (1-\alpha)\Pi^1(q^1) + \alpha\Pi^0(q^0) + \alpha\int_{q^0}^{q^1}(c-\bar{c})$$

$$\text{Subject to } : q^0 \le q^1 \le q^0 + U$$

Note that only the purple area is taxed. Hence, given q^1, the best choice of q^0 is the one so that the price remains the same, since it would lead to a nil purple area. Then, the profit will simply be $\Pi^1(q^1)$. The optimum value for q^1 is then the same as in the problem in section 3.

In the end, the dispatch will be the same but there will be no tax.

Now, if demand is uncertain, the problem becomes very difficult. Thus, we will not try to solve it but we may assume intuitively that we will probably not obtain an offer function constant between expected demand and expected demand incremented by line capacity, especially if the standard deviation of demand is higher than line capacity.

Still, we may assume that companies will announce higher costs than they would have, had there been no such line allocation. Although the dispatch would not change much, taxes would be quite low.

Chapter 9

Statements about this internship

This chapter has no scientific interest. It is a personnal reflection about my skills and my weaknesses, that this internship has enhanced.

9.1 Working skills

During this project, I have had the chance to develop and improve my mathematical skills. I have been amazed by the necessity for me to use plenty tools that I have learned at the Ecole Polytechnique in the last months before the internship. The course of Operations Research by Dr. Bonnans and Dr. Gaubert and the course of Game Theory by Dr. Sorin have turned out to be very useful. I have enjoyed applying them. I have also had the chance to program in Ampl/CPLEX.

Beside pure science, my general knowledge has been increased as I have read a lot about electricity markets all over the world. The fact that their liberalization is very recent (about two decades old for most countries) and that their design is in constant evolution makes their study very interesting to study. In particular, the European transmission network management is a burning issue.

I have taken pleasure in working in cooperation with my supervisor. Discussions about the work have always been very fascinating. But I have also appreciated my flexibility and my freedom in my work, which have given me the chance to take initiatives, to make attempts and to follow through my ideas.

9.2 Human skills

However, what I have appreciated the most during this internship is the opportunity to live and work in New Zealand. This trip was a tremendous challenge for me, as I was leaving to a country at the other end of the world, in which I knew nobody. My only contact was my supervisor's email address. Before the trip, I was really afraid to be lost and lonely far away from my friends and family, in this English-speaking country that possesses its own culture.

Despite my fears, I have enjoyed this place a lot. I have particularly loved the international and multicultural environment, where, at lunch for instance, I was surrounded by people from diverse countries. Here is a little list of nationalities of people I have met : Irish, Dutch, Canadian, Danish, Brazilian, Singaporean, Chinese, South Korean, Indian, German, English, Chilean, Iranian, Colombian, Belgian. Do not forget about the local Kiwis, and those ever-present French people. In particular, discussions about the football world cup were very exciting, despite the French disaster. This experience makes me want to travel even more, and to work in very international places.

In this cosmopolitan environment, I have had the chance to practise English more than ever before. Although I have had huge difficulties to understand the Kiwi accent and to take part in conversations at first, I now really enjoy talking the language of Shakespeare. Not only have I intervened in group discussions, but I have also managed to tell stories and make jokes, which has been a massive step for me.

Furthermore, I have been kind of surprised to meet 30-year-old students achieving their masters' degree. However, I have found their lifestyle very interesting, and I have realised that there is no age limit to start working in a company. In particular, their attitudes have inspired me and incited me to think about extending my studies. I now consider heading for a Ph. D.

9.3 Conclusion about my personnal reflection

After 11 weeks in Auckland, I am definitely feeling at home here. I feel that I have succeeded both in achieving my research project and in settling well in this city. The fact that I have overcome my fears and that I have risen to the challenge has given me plenty of confidence for upcoming projects.

I am definitely looking forward to the next step of my studies, my Master's Research at the Ecole Polytechnique of Montreal.

Chapter 10

Conclusion

In this paper, we have mainly studied an alternative to the current HVDC line allocation. In a first part, we have made hypotheses on the electricity network to simplify the equations and the study of the game problem of the alternative allocation.

Then, in a naive first study, we have seen the possibility of unfairness and inefficiency. It actually occurs when line rights are not delivered to the right firms and the right plants.

With another simplified model, we have shown the incentives for a competitive company to follow malicious strategies. The reason of that is the possibility of increasing the southern price by the use of its supply function. Therefore, it is ready to sacrifice important outputs to increase the price, and his profits.

The weirdest part of that is that the second player actually take advantage of that, not only because of the fact that by sacrifying some outputs the first company allows the second to produce, but also because the increase of price is profitable for the second company too. In fact, even when the number of companies is higher, a coalition that would sacrifice the profits of player 2 to increase the southern price would maximize the cumulated profits of all companies. Such behaviours must be feared.

What happens with the introduction of a spot market is an increase of choices for companies to set a developed strategy. This leads more easily to market power.

In the problem with certain demand, we have shown that this leads to inefficiency and market power. Furthermore, the optimal strategies for companies are mixed strategies. Therefore, datas of the market become uncertain, and the incomes for the System Operator are not guaranteed. Instability occurs. Moreover, the high and uncertain spot prices would be a wrong signal for companies who would like to enter the market. They should know that by entering the market, they would probably deregulate the equilibrium and possibly greatly lower the price.

When uncertainty on demand is introduced, Nash Equilibria remain similar. The existence of two polar strategies - setting the price or letting the price being set - leads to these mixtures that jeopardize the system with instability, inefficiency and market power.

The pro-rata-with-incremental-profit allocation model we have introduced is not much better, as companies are still incited to follow strategies that prevent them from paying most of taxes. Although market power diminishes, companies are still tempted to give very high announced costs. The line operator revenues would probably not be high enough.

To conclude, the results of this paper suggest that the proposals that have been made would lead to instability, inefficiency and market power, due to strategic behaviours. The criticisms that could be imputed to them are much more serious than those of the current HVDC line allocation. In order to respond to these criticisms, taxing northern consumers, who benefit from the line but do not pay, as well as the southern generators may be considered.

.1 Appendix 1 : Source code of figure 7.9

```
* SFE model
*
* Written by Andy Philpott, based on Xinmin Hu's model}
* Assume a discretization of demand shock}
* Gives a supply-function response to beta}
* Note: Assumes symmetric SFE}

SETS
i player  / 1,  2 /
k discretization /1*51/;
;

scalar delta   >0 to make constraints strict
/0.005/ ;

scalar r /3/ ;
*scalar pmax   price cap
*/5.0/ ;
*scalar pmin   price floor
*/0.01/ ;

parameter c(i)  cost coefficient   /
1  1
2  5      /         ;

parameter d(k) demand shock /
1        0.00
2        0.02
..  ..
51       1.00
/       ;

*\$ontext
parameter q0(i,k) ;
parameter p0(k);

*q0(i,k) = d(k)/2;
*p0(k)=  3 + (d(k)/2)*log(d(k)/2) - 5.307*d(k)/2;
*\$offtext

positive variable
q(i,k) quantity offered by player i at demand k
pi(k)  clearing price at demand level k
;

variable
s(i,k) slope of other player
t(i,k) slope of player i
pt(k)
u(k)  error terms
error
;

equation
objective
response(i,k)
demand(k)
```

```
stdef1(k)
stdef2(k)
decreasing1(i,k)
decreasing2(i,k)
decreasing3(k)
tilde(i,k)
monotonic1(k)
monotonic2(k)
monotonic3(i,k)
*symmetry(k)
;


objective .. error =e= 1000*sum(k,u(k)*u(k));

response(i,k) .. s(i,k)*(r - pi(k) - c(i)*q(i,k)) + q(i,k)  =e= 0;
demand(k).. sum(i, q(i,k)) + u(k) =e= d(k) ;

stdef1(k).. t('1',k) =e= s('2',k) ;
stdef2(k).. s('1',k) =e= t('2',k);

decreasing1(i,k).. t(i,k) =l= 0;
decreasing2(i,k)\$(ord(k) lt card(k)).. q(i,k+1)- q(i,k) =g= 0;
decreasing3(k)\$(ord(k) lt card(k)).. pi(k+1) - pi(k)  =l= 0;

tilde(i,k)\$(ord(k) lt card(k)).. q(i,k+1) - q(i,k) - t(i,k+1)*pi(k+1)
           + t(i,k)*pi(k) + (t(i,k+1)-t(i,k))* pt(k) =e= 0;
monotonic1(k).. pi(k) - delta =g=  pt(k) ;
*monotonic1a(k)\$(ord(k) lt card(k)).. pt(k) + delta =l=  pi(k+1) ;
monotonic2(k)\$(ord(k) lt card(k)).. pt(k) - delta =g= pi(k+1)  ;
monotonic3(i,k)\$(ord(k) lt card(k)).. q(i,k) + delta =l=  q(i,k+1) ;
*symmetry(k).. q('1',k) =e= q('2',k);


model sfe /all / ;

option nlp=conopt ;

option decimals = 8;

t.up(i,k)= -0.001 ;
s.up(i,k)= -0.001 ;
t.lo(i,k)= -5.0 ;
s.lo(i,k)= -5.0 ;
*q.l(i,k) = q0(i,k);
*pi.l(k)\$(ord(k) lt card(k)) = p0(k);
*pi.up(k)\$(ord(k) lt card(k))= p0(k)+0.2;
*pi.lo(k)\$(ord(k) lt card(k))= p0(k)-0.2;
q.up(i,'1')= 0;
q.lo(i,'1')= 0;
pi.up('51')=0.000;
pi.lo('51')=0.000;
pi.up('1')=3.000;
pi.lo('1')=3.000;


solve sfe minimising error using nlp;


*q.up(i,k)= q.l(i,k)+ 0.1;
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```
*q.lo(i,k)= q.l(i,k)- 0.1;
*c('2') = 2.5;
*solve sfe minimising error using nlp;
*pi.up(k)\$(ord(k) lt card(k)) = pi.l(k) + 0.2;
*pi.lo(k)\$(ord(k) lt card(k)) = pi.l(k) - 0.2;
c('2') = 3.0;

solve sfe minimising error using nlp;
*pi.up(k)\$(ord(k) lt card(k)) = pi.l(k) + 0.2;
*pi.lo(k)\$(ord(k) lt card(k)) = pi.l(k) - 0.2;
q.up(i,k)= q.l(i,k) + 0.1;
q.lo(i,k)= q.l(i,k) - 0.1;
c('2') = 4.0;


solve sfe minimising error using nlp;
*pi.up(k)\$(ord(k) lt card(k)) = pi.l(k) + 0.4;
*pi.lo(k)\$(ord(k) lt card(k)) = pi.l(k) - 0.4;
q.up(i,k)= q.l(i,k) + 0.1;
q.lo(i,k)= q.l(i,k) - 0.1;
c('2') = 5.0;
*pi.up('50')=0.000;
solve sfe minimising error using nlp;



display q0;
display p0;

FILE RES /D:\docs\2010\KimFrew\sfeLEN50.out/;
PUT RES;
RES.nd=5;
RES.nw=10;
RES.ap=0;

put "Quantity1   Quantity2     Price     PTilde        " / ;
loop (k,
put q.l('1',k);
put "  ";
put q.l('2',k);
put "  ";
put pi.l(k)
put "  ";
put pt.l(k)
/;
)   put /;
```

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