Derivation of a Prime verification formula to prove the related open problems

Abstract
In this document, we will develop a new formula to calculate prime numbers and use it to discuss open problems like Goldbach, Polignac and Twin prime conjectures, perfect numbers, the existence of odd harmonic divisors, ...

Key-words
Goldbach's conjecture; prime; twin prime conjecture; Polignac; Carl Pomerance; Mersenne prime; Euler; Euclid; divisor function; aliquot; perfect number; helix; odd and even harmonic divisor; ore number; complex plane.

Introduction
The following document originated during our study of primes.

We tried to create a schematic representation of the mechanism that lead to the creation of primes. Following, we tried to mathematically describe this graphical representation. It resulted in a new formula to calculate if a number is prime or not.

We guessed that our new formula could be useful in understanding the mechanism of primes and possibly in proving some open questions.

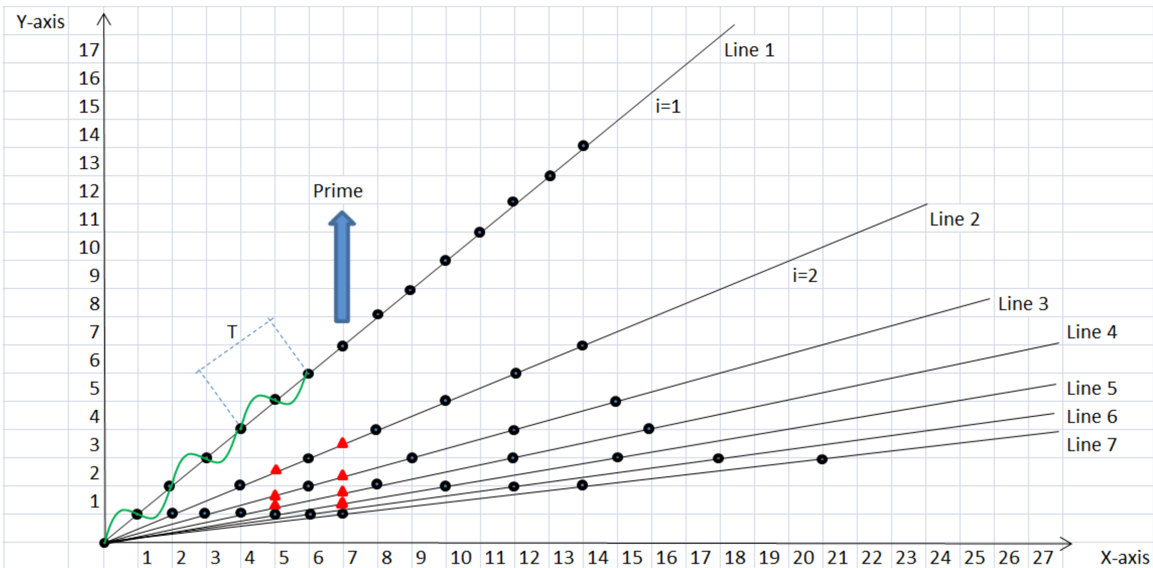
After primes, we read about divisibility and open problems related to that subject and found that we were on the right track to discuss these problems as well.

After deriving our formula, we will discuss about:
- Goldbach's conjecture: "Every even integer greater than 2 can be expressed as the sum of two primes."
- Polignac's conjecture: "For any positive even number n, there are infinitely many prime gaps of size n. In other words: There are infinitely many cases of two consecutive prime numbers with difference n."
- Twin Prime conjecture: "for every natural number k, there are infinitely many prime pairs p and p′ such that p′ – p = 2k. The case k = 1 is the twin prime conjecture."
- Perfect numbers: "a perfect number is a positive integer that is equal to the sum of its proper positive divisors excluding the number itself (also known as its aliquot sum)."
- Harmonic divisor numbers: "Ore conjectured that no odd harmonic divisor numbers exist other than 1."

Methods & Techniques

Step 1: development of a new formula to check if a number is prime or not

We will refer to the following picture as a graphical aid to derive our formula:



Let's draw an X&Y axis with on:
- X-axis: the natural numbers n of which we want to calculate if they are prime or not
- Y-axis: the divisors i of the corresponding natural number n on X

Suppose, we want to draw the line that shows us all the numbers that can be divided by 1. In that case, we draw a line that starts at (0,0) and that goes through (1,1). We will mark all the valid divisors on the line by a full dot. Since every natural number can be divided by 1, we can mark the following dots on line 1: (1,1), (2,2), …(n,n), ... So, all possible dots are marked on that line.

Suppose, we want to draw the line that shows us all the numbers that can be divided by 2. In that case, we draw a line that starts at (0,0) and that goes through (2,1). Only the even numbers will be divisible by 2. Hence we can mark the following dots on line 2 in full: (2,1), (4,2), …, (n,n/2), …

Suppose, we want to draw the line that shows us all the numbers that can be divided by i (ϵN). In that case, we draw a line that starts at (0,0) and that goes through (i,1).
The full dots that represent the divisors by i will again be on equal distant and can be marked on line i as: (i,1), (2.i,2), … (n.i,n), …

This line will have the following formula for line i:

$$y = \frac{1}{i}.x$$

After we draw a few lines and highlight the divisors with full dots, we can easily see that the prime numbers will be these x=n that have only a full dot on line 1 and on line n and not on the lines i from 2 to n-1. We could mark these couples with a different sign (triangle).

We can now "sew" the X,Y plane starting from the origin through the full dots, that represent the divisors of the natural numbers on the X-as. We will sew the X,Y plane on each line along the Z axis using a sinus function with following characteristics:

Period T of sinus function covering 3 dots:

$$T = 2.\sqrt{i^2 + 1^2} = 2.\sqrt{i^2 + 1}$$

Hence we get for our sinus function:

$$z(d) = \sin\left(\frac{2\pi}{T}.d\right) = \sin\left(\frac{2\pi}{2\sqrt{i^2+1}}.d\right) = \sin\left(\frac{\pi}{\sqrt{i^2+1}}.d\right)$$

With d denoted as the distance we have traveled on the line i while sewing the dots.
This can be expressed as:

$$d = \sqrt{x^2 + y^2}$$

Note: our sinus will become zero at each cross through the X&Y-plane on a full dot.

Working out the above, we get:

$$z(x) = \sin\left(\frac{\pi\sqrt{x^2 + y^2}}{\sqrt{i^2+1}}\right) = \sin\left(\frac{\pi\sqrt{x^2 + (\frac{x}{i})^2}}{\sqrt{i^2+1}}\right) = \sin\left(\frac{\pi x}{i}\right)$$

Natural numbers n that are not prime will have at least one line i with a full dot that will represent a divisor different than 1 or itself and hence an i between 1 and n where the above sinus will be zero. This will not be the case for prime numbers.

Hence we can state the formula for determining if a natural number is prime or not as:

$$P(n) = \prod_{i=2}^{n-1} \sin\left(\frac{\pi n}{i}\right) <> 0 => n = \text{Prime}$$

(formula 1.)

In literature, it is said that there is no known useful formula that sets apart all of the prime numbers from composites. This proves then the contrary.

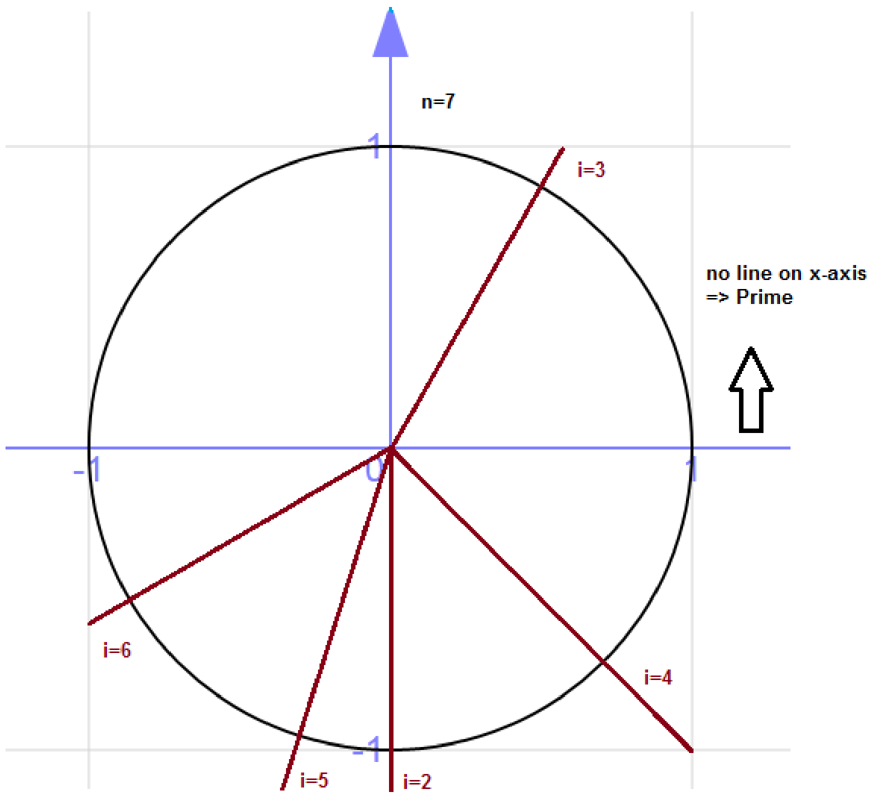
If P(n)=0, this means that at least one divisor was found between 1 and n and hence that n is not prime.

The meaning of this formula can be graphically represented by drawing vectors from (0,0) in the direction of the different angles $\frac{\pi n}{i}$ for i=2 to n-1.

We will show the main different cases with an examples:

<u>Example 1:</u> Prime number (example n=7):

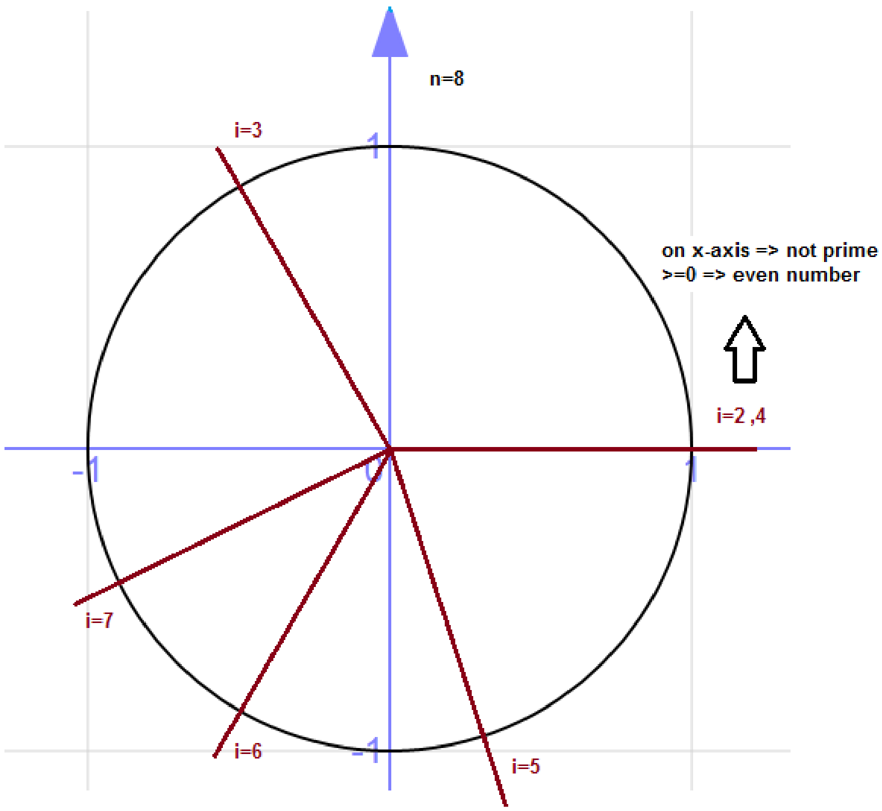
n	7						
i	1	2	3	4	5	6	7
angle	1260	630	420	315	252	210	180
mod(angle,360)	180	270	60	315	252	210	180



There is no vector along the x-axis (if we consider only n=2 to n-1), so n is a prime number.

Example 2: even non Prime number with only even divisors (example n=8):

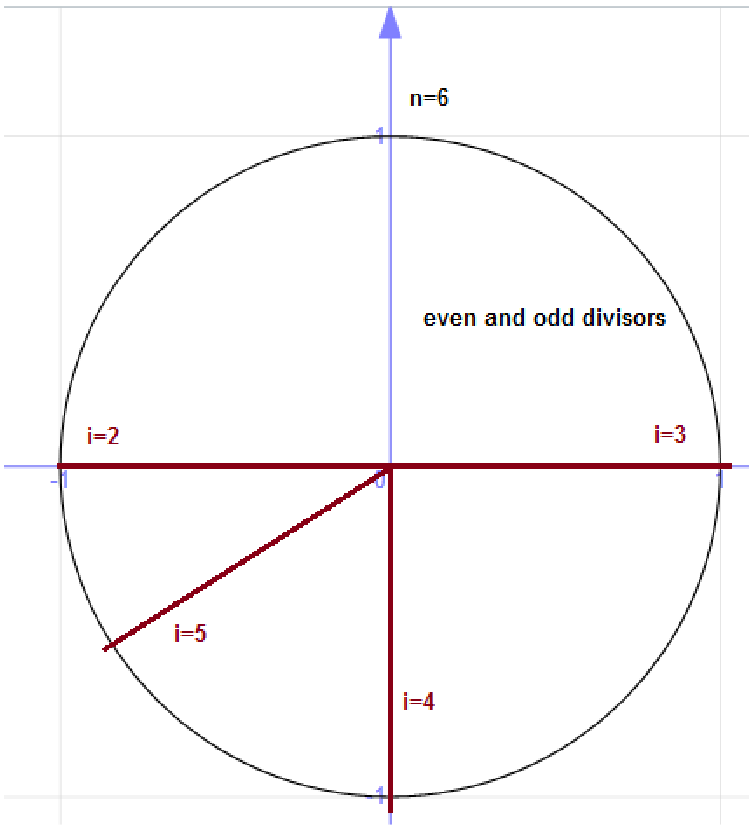
n	8							
i	1	2	3	4	5	6	7	8
angle	1440	720	480	360	288	240	206	180
mod(angle,360)	0	0	120	0	288	240	206	180



There is at least one vector along the x-axis and it is pointed to the right.

Example 3: even non Prime number with odd and even divisors (example n=6):

n	6			$\sigma_1(n)$	4			$\sigma_1(n)$-n	6	aliquot sum	
i (or di)	1	2	3	4	5	6					
degrees	1080	540	360	270	216	180					
mod(degrees,360)	0	180	0	270	216	180					

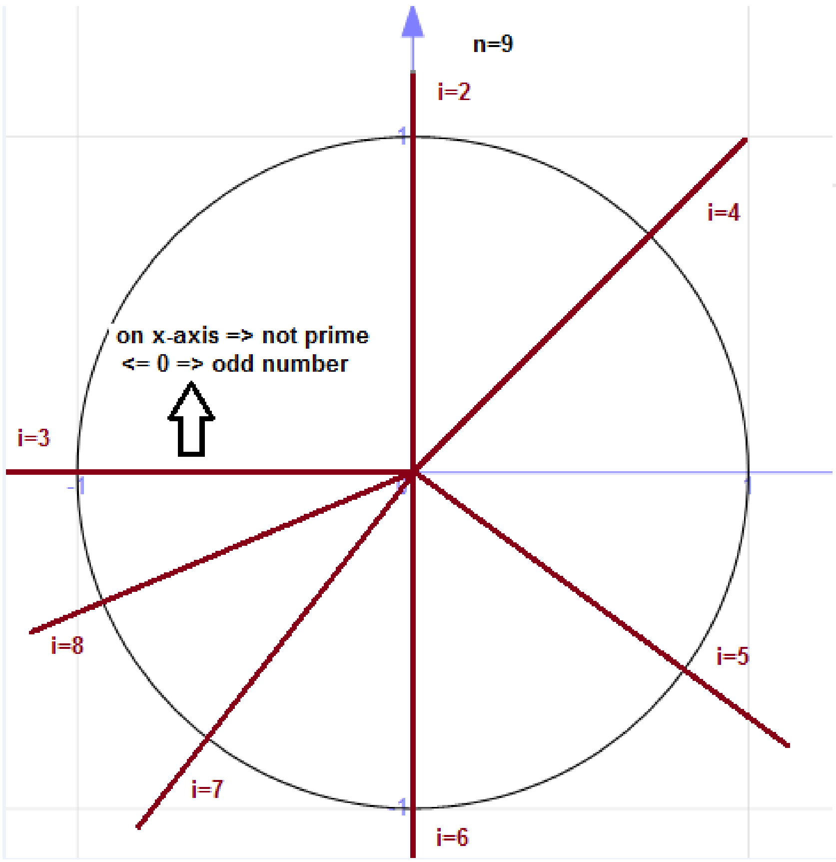


There is more than one vector along the x-axis and they are pointed to the left and to the right.

Attention: 6 is also a perfect number. This means that the sum of its aliquot divisors equals itself (1+2+3=6). Unfortunately, this can not immediately be seen in the graphic.

Example 4: odd non Prime number (example n=9):

n	9								
i	1	2	3	4	5	6	7	8	9
angle	1620	810	540	405	324	270	231	202.5	180
mod(angle,360)	180	90	180	45	324	270	231	203	180



There is at least one vector along the x-axis and it is pointed to the left.

Out of the above graph follows that as soon as i>n/2, there is no more chance of a vector hitting the 180° line as it's angle will always be between >180° and <360°.

It is clear from the above graphical representations of our formula 1 that every vector set out starting from 2, 3, … will or will not hit the 180° line. The first vector δ that will hit the 180° line (for i= δ) will determine n as a non prime. δ will then be the first divisor of n.

It is clear that if δ is the first divisor of n then n- δ will be a multiple of δ as well. Since it is a multiple it cannot be a prime. If δ is the first divisor then it follows that δ-1 will not be a divisor. Hence δ-1 is a relative prime. Following the logic, it also follows that n- δ+1 will be a relative prime. The opposite is also true: if δ is not a divisor of n then n- δ will also not be a multiple of δ as well in n.

In the case when there is no vector that hits the 180° line, n will be a prime number. When we choose the variable n to be 2n, then it is obvious that there will always be a vector that hits the 180° line before it reaches 2n-1. Would this not be the case then 2n would be a prime which is a contradiction since it is even.

It is clear that we already proved Golbach's conjecture out of the above.

Remark 1:

The above graphical representations can be interpreted as a spring that is twisted in counter clockwise direction for a maximum number of turns for i=2 and then slowly released in clockwise direction in ever smaller steps until i=n-1.

Actually for better understanding we can say that the spring has been twisted n turns (nx360°) and released till 180° (in ever smaller steps) but we won't take into account the first and the last position (i=1 and i=n) for calculating primes.

The angle for an arbitrary i is: $\varphi_i = \frac{\pi n}{i}$

The step taken between i-1 and i equals:

$$\Delta\varphi_i = \frac{\pi n}{i} - \frac{\pi n}{i-1} = \frac{-\pi n}{i(i-1)}$$

The smallest step will be the last one for i=n-1:

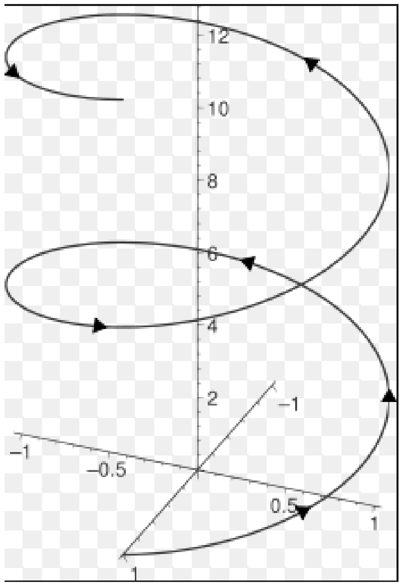
$$\Delta\varphi_{n-1} = \frac{\pi}{n-1}$$

At that moment, the vector will be the closest to 180° but not reaching it:

$$\varphi_{n-1} = \frac{\pi n}{(n-1)}$$

So this last vector will not contribute to n being not prime. In fact none of the vectors where i>n/2 will contribute to n being not prime.

In graphical representations, it sometimes looks like the vectors are randomly distributed but this is because they take bigger jumps for smaller i and that way they look like they have mixed up. But if we would represent it as being on a helix, things would become more obvious. So the previous examples can be seen as the projections on the xy-plane of what happens in reality on a helix.



Remark 2:

For i=1, the vector representing $\frac{\pi n}{i}$ will be on positive x-axis for even n and on negative x-axis for odd n. This is mentioned here because this vector is not drawn in the above representations but it might have a use to check if n is a perfect number.

Carl Pomerance has presented a heuristic argument which suggests that no odd perfect numbers exist.

So for perfect numbers, the vectors will start on positive x-axis and end on negative x-axis.

Each time that a vector hits the x-axis, it seems that i is a divisor of n.

Remark 3:

Using Euler’s formula, we could also write:

$$P(n) = \prod_{i=2}^{n-1} Im(e^{i\left(\frac{\pi n}{i}\right)}) <> 0 => \text{Prime}$$

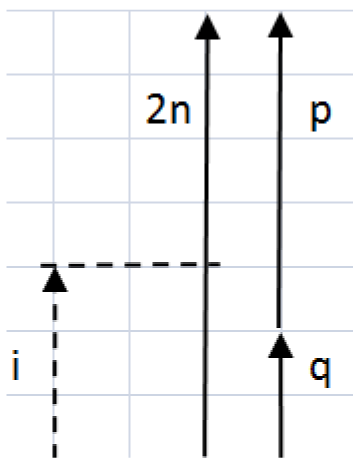
This means as well that if we set out the vectors for n in the direction $\frac{\pi n}{i}$ for i=2 to n-1 and one of these vectors is entirely on the x-axis then n is not a prime.

Step 2: splitting an even number into the sum of 2 primes

Goldbach's conjecture states that every even number can be written as the sum of two primes.
We can agree on following convention:

2n	every even number
q	prime number greater than 2 (and q≤p)
p	prime number smaller than 2n-2

With: 2n=p+q



So for 2n, p and q, we will be able to write:

$$P(2n) = \prod_{i=2}^{2n-1} \sin\left(\frac{2\pi n}{i}\right) = 0 => not \text{ Prime}$$

This is obvious since for i=2, the sinus term will equal 0.

$$P(p) = \prod_{i=2}^{p-1} \sin\left(\frac{\pi p}{i}\right) <> 0 => \text{Prime}$$

$$P(q) = \prod_{i=2}^{q-1} \sin\left(\frac{\pi q}{i}\right) <> 0 => \text{Prime}$$

The above 2 equations are also obvious since a prime number is not divisible, hence the argument will never be a multiple of π.

Moreover, if we find one of the 2 primes (p or q) then it follows automatically that the other one (2n-p or 2n-q) is also prime:

$$P(2n-q) = \prod_{i=2}^{2n-q-1} \sin\left(\frac{\pi(2n-q)}{i}\right)$$
$$= \prod_{i=2}^{2n-q-1} \left[\sin\left(\frac{2\pi n}{i}\right)\cos\left(\frac{\pi q}{i}\right) - \sin\left(\frac{\pi q}{i}\right)\cos\left(\frac{2\pi n}{i}\right)\right] <> 0 => \text{Prime}$$

The above is true because the sinus in the first term will become 0 for i=2 and the second term will never become 0.

$$P(2n-p) = \prod_{i=2}^{2n-p-1} \sin\left(\frac{\pi(2n-p)}{i}\right) <> 0 => \text{Prime}$$

The above is true because of the same reasons.

This means that we have to find only one prime below each even number as the other prime will follow from the equation.

Since 2 is the first prime and 2 is even, it follows that we will have to start looking for the first prime >= 3.

Summary, if we can prove that there always exists a prime under following condition:

3<=q<=n(=2n/2)

Then we have proven Goldbach's conjecture.

From the above, it is obvious that we have to consider only even numbers where 2n>=6.

As 3 is the first prime >2 that fulfills the requirement, it follows that we have proved Goldbach's conjecture.

Step 3: relationship between the sum of primes

From the above formula’s, we can deduct an interesting relationship:

$$P(2n) = \prod_{i=2}^{2n-1} \sin\left(\frac{\pi(p+q)}{i}\right) = \prod_{i=2}^{2n-1} \left[\sin\left(\frac{\pi p}{i}\right)\cos\left(\frac{\pi q}{i}\right) + \sin\left(\frac{\pi q}{i}\right)\cos\left(\frac{\pi p}{i}\right)\right]$$

Since $P(2n) = 0$ and $\sin\left(\frac{\pi p}{i}\right) <> 0$ as well as $\sin\left(\frac{\pi q}{i}\right) <> 0$, we have a linear equation that will need to have solutions for $\cos\left(\frac{\pi p}{i}\right) <> 0$ as well as $\cos\left(\frac{\pi q}{i}\right) <> 0$.

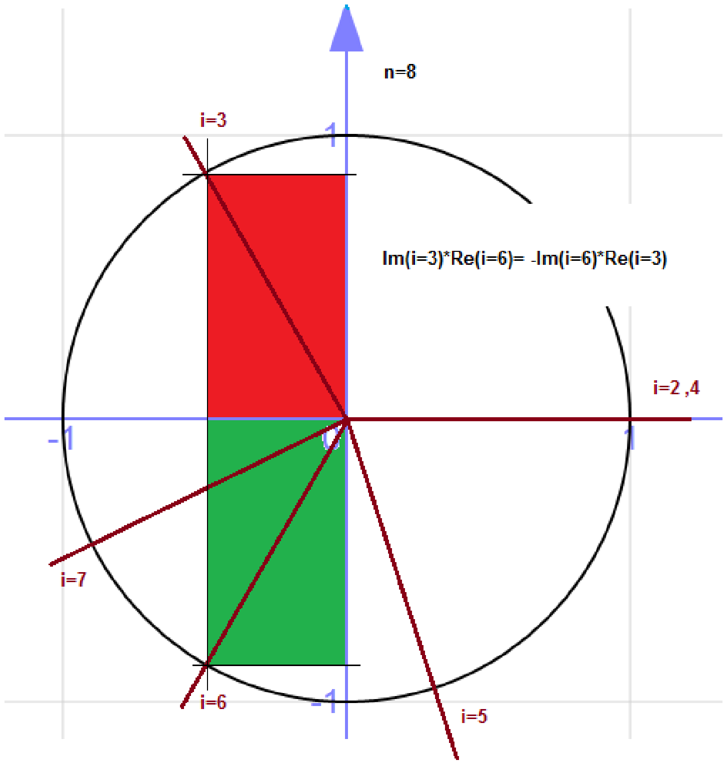
This was to be expected since our formula for calculating if a number is prime is nothing more as a consecutive multiplication of vectors until one multiplication equals zero as a result whereby we know that n is not a prime.

In formula’s:

$$(a+bi)(c+di) = (ac-bd) + (bc+ad)i$$

The imaginary part is 0 if $bc = -ad$

This is illustrated in the example below:



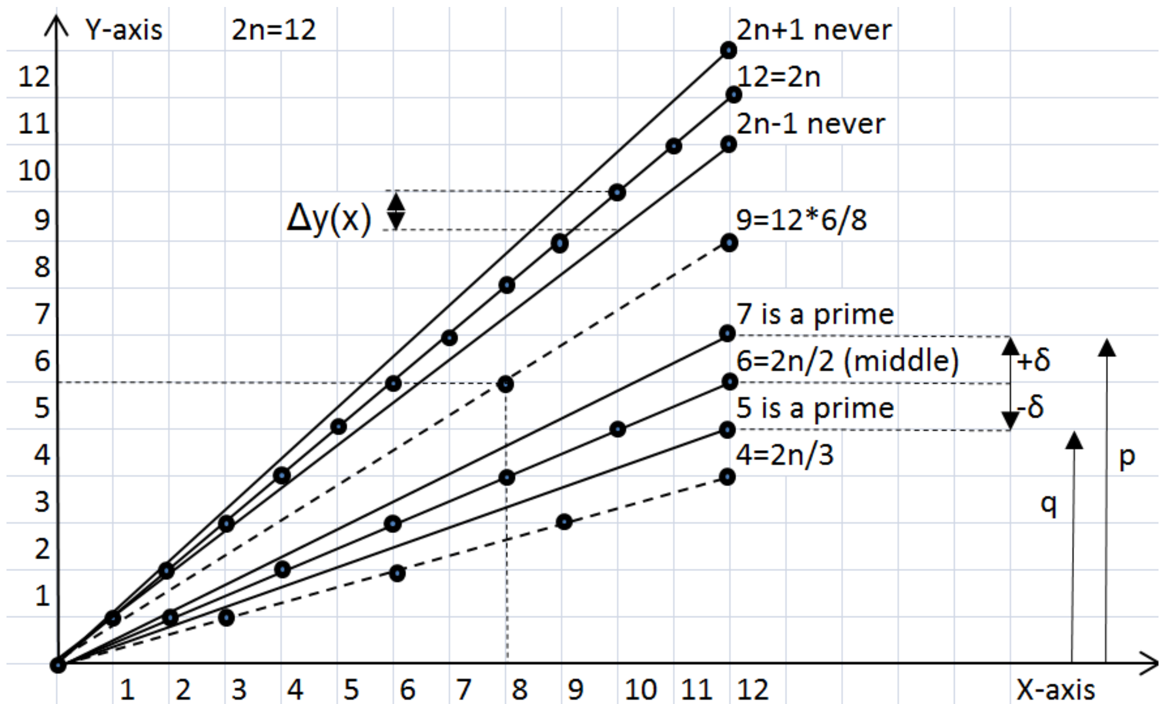
This means that out of 2 prime numbers (or vectors), a non prime number (or vector) can come if the surface made from the imaginary part of the first vector with the real part of the second vector is the same but opposite as the surface made from the imaginary part of the second vector with the real part of the first vector.

The above can be shown in a small example demonstrating the sum of 2 primes 7 and 11. One can see in the picture that the 2 factors are equal but opposite for i=3, 6 and 9 resulting in a non prime sum:

	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG
55																			
56		2n	18		2n=p+q														
57		p	7		formula used for R60: =SIN(PI()*$Q57/R59)*COS(PI()*$Q58/R59)+SIN(PI()*$Q58/R59)*COS(PI()*$Q57/R59)														
58		q	11																
59		i		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
60		sin(pi*(p+q)/i)		0	-0	1	-0.95	0	0.97	0.71	-0	-0.59	-0.91	-1	-0.94	-0.78	-0.5878	-0.3827	-0.1837
61		sin(pi*p/i)cos(pi*q/i)		0	0.43	0.5	-0.77	-0.43	0	-0.15	-0.49	-0.77	-0.91	-0.93	-0.88	-0.78	-0.6655	-0.5449	-0.4287
62		cos(pi*p/i)sin(pi*q/i)		0	-0.43	0.5	-0.18	0.43	0.97	0.85	0.49	0.18	-0	-0.07	-0.06	0	0.0777	0.1622	0.245

Step 4: graphical representation of divisibility and prime

We will refer to the following picture as a graphical aid to talk about divisibility:



In the above picture we will draw a line from (0,0) to (2n,2n) as a starting point.
We notice that this line goes through all the couples (i,i) from i=0 to 2n.
This means that 2n is divisible by 2n under all circumstances.
We will mark all integer couples with a dot.

If we want to know if 2n is divisible by a certain number q (ex. q=4), we draw a line from (0,0) to (2n,q) as well. If the line hits one or multiple times couples (i,j) with i,j between 0 to 2n, we know that 2n is divisible by q.

In fact, any line between (0,0) and (2n,q) that hits integer couples (i,j) shows that there is a linear relationship between q and 2n using only integers. The relationship is: $q=2n*j/i$.

One of the interesting matters is that the 2n-1 line (as well as the 2n+1 line) will never hit any integer (i,j) except (0,0) and (2n,2n-1). This is because $0 \le \Delta y(x) < 1$ for all $x<2n$. In fact $\Delta y(x)=1*x/2n$. So, it doesn't hit an integer value anymore before it reaches x=0. Unfortunately, 1 is not considered a prime number otherwise Goldbach's conjecture would also be proved by this.

But what it does show is that for every 2n (which is obviously not a prime), 2n+1 and 2n-1 will not have a linear relationship in the form of q=2n*j/i with ni. Therefore 2n-1 will either be a prime or a composite of primes.

Sometimes, n=2n/2 happens to be a prime number. In that case, it is obvious that 2n can be split into n and n as prime numbers.

For the other case (where n=2n/2 ≠ prime), we claim that every line that goes through (0,0) and (2n,n+δ) with δ an integer ≥0 and ≤ n-3, this line together with the line that goes through (0,0) and (2n,n-δ) with δ an integer ≥0 and ≤ n-3 will equal to the line that goes through (0,0) and (2n,2n). This means that if we find a δ where n- δ=q represents a prime line, then the other line n+ δ=p will also represent a prime line. Both lines will represent the proof of Goldbach's conjecture.

We will now use formula 1 from above to verify our findings:

<u>for 2n:</u>

$$P(2n) = \prod_{i=2}^{2n-1} \sin\left(\frac{2\pi n}{i}\right) = 0 => 2n \neq \text{Prime}$$

This is because for i=n, sin(2π) will equal to 0 (as well as for i=2).

<u>for n:</u>

$$P(n) = \prod_{i=2}^{n-1} \sin\left(\frac{\pi n}{i}\right) = 0 \; or \; \neq 0 \;?$$

The above =0 if one of i (between 2 and n-1) is a divisor of n. This is the case if on the line from (0,0) to (2n,n) on the above drawing there is one or more dots marked. If the above formula doesn't equal 0 then n is the prime number also mentioned above.

In the case where n is not the prime we are looking for, then we will look for a prime, a little higher or lower than n.

<u>for n$\pm\delta$:</u>

$$P(n \pm \delta) = \prod_{i=2}^{n\pm\delta-1} \sin\left(\frac{\pi(n \pm \delta)}{i}\right) = 0 \; or \; \neq 0 \;?$$

using the following trigonometric identity:

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)sin(\beta)$

We get:

$$P(n \pm \delta) = \prod_{i=2}^{n\pm\delta-1} \left[\sin\left(\frac{\pi n}{i}\right). \cos\left(\frac{\pi\delta}{i}\right) \pm \cos\left(\frac{\pi n}{i}\right). \sin\left(\frac{\pi\delta}{i}\right) \right]$$

The first term will be 0 since n was not a prime (and hence divisible by i) but if we choose δ such that it is not divisible by i, then $n \pm \delta$ will seem to be one of the primes that we were looking for. The other one can be found by subtracting the first one from 2n as shown before.

<u>Case for n-δ:</u>

$$P(n-\delta) = \prod_{i=2}^{n\pm\delta-1} \sin\left(\frac{\pi(n-\delta)}{i}\right) = 0 \; or \; \neq 0 \; ?$$

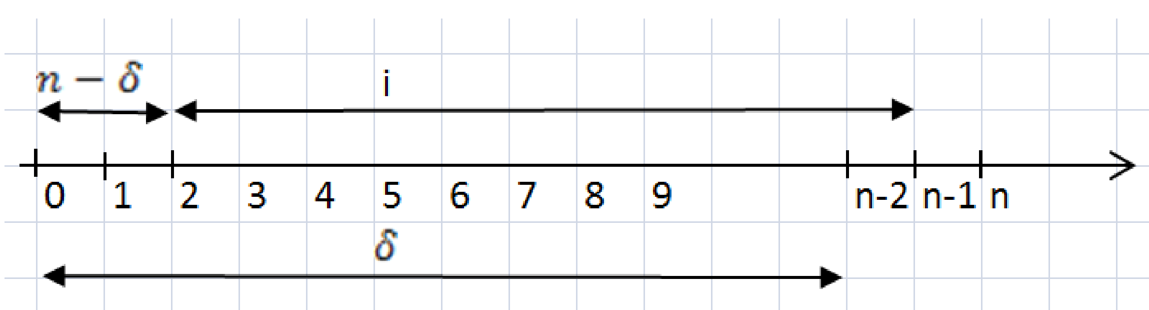
The above equation is ≠ 0 if:

$$\frac{(n-\delta)}{i} \neq \; 1 \; or \; more \; generaly \; \neq k$$

Or if:

$$\text{n-δ} \neq \; i \; or \; more \; generaly \; \neq k\text{i}$$

Because δ can go from 0 to n-2 and i can go from 2 to n-1, it follows from the below picture that n-δ ≠ i.



The above as well already illustrates the sum of the twin primes.

Step 5: Polignac’s conjecture

Polignac’s conjecture states: “For any positive even number n, there are infinitely many prime gaps of size n. In other words: There are infinitely many cases of two consecutive prime numbers with difference n.”

To highlight that n is even, we will write n as 2n so that 2n=p-q.

To illustrate our proof, we will show an example below:

	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA
41													
42		2n	12		2n=p-q								
43		p	23		formula used for R46: =SIN(PI()*$Q43/R45)*COS(PI()*$Q44/R45)-SIN(PI()*$Q44/R45)*COS(PI()*$Q43/R45)								
44		q	11										
45		i		2	3	4	5	6	7	8	9	10	11
46		sin(pi*(p+q)/i)		-0	-0	0	0.95	-0	-0.78	-1	-0.87	-0.59	-0.28
47		sin(pi*p/i)cos(pi*q/i)		0	-0.43	0.5	0.77	-0.43	-0.17	-0.15	-0.75	-0.77	-0.28
48		cos(pi*p/i)sin(pi*q/i)		0	-0.43	0.5	-0.18	-0.43	0.61	0.85	0.11	-0.18	0

Using the previous formula’s, this would mean that we are looking for:

$P(2n) = 0$; $P(p) \neq 0$; $P(q) \neq 0$

Let’s take any prime number (out of the infinite amount of existing primes) and call it p if it is >2n or call it q if it is <2n.

Suppose, we call it p then:

$$P(p) = \prod_{i=2}^{p-1} \sin\left(\frac{\pi p}{i}\right) = \prod_{i=2}^{p-1} \left[\sin\left(\frac{\pi(2n+q)}{i}\right)\right] \neq 0 \; since\ p\ is\ prime$$

$$P(p) = \prod_{i=2}^{p-1} \left[\sin\left(\frac{2\pi n}{i}\right)\cos\left(\frac{\pi q}{i}\right) + \cos\left(\frac{2\pi n}{i}\right)\sin\left(\frac{\pi q}{i}\right)\right] \neq 0$$

However the first term in the equation is 0 because $\sin\left(\frac{2\pi n}{i}\right) = 0$.
This means that the second term cannot be 0.
This means that $\sin\left(\frac{\pi q}{i}\right) \neq 0$ or that q is also prime.

This means that for every prime, we can find a corresponding prime such that 2n=p-q. Since there are an infinite amount of primes to choose from at the start, it means that we have proven Polignac's conjecture.

Step 6: Twin primes

The Greeks speculated that there are an infinitude of twin primes. But they weren't able to prove that.

Using the previous formula's, this would mean:

$P(2n) = 0$; $P(2n+\delta) \neq 0$; $P(2n-\delta) \neq 0$

$$P(2n) = \prod_{i=2}^{2n-1} \left[\sin\left(\frac{\pi 2n}{i}\right)\right] = 0$$

$$P(2n-\delta) = \prod_{i=2}^{2n-\delta-1} \left[\sin\left(\frac{\pi(2n-\delta)}{i}\right)\right]$$
$$= \prod_{i=2}^{2n-\delta-1} \left[\sin\left(\frac{\pi 2n}{i}\right).\cos\left(\frac{\pi\delta}{i}\right) - \cos\left(\frac{\pi 2n}{i}\right).\sin\left(\frac{\pi\delta}{i}\right)\right] \neq 0$$

The first factor in this equation will be 0 because P(2n)=0.
The second factor will never be 0 because the cosines argument will never reach π/2 and the sinus argument will never reach π.
Hence P(2n- δ) will never be 0.

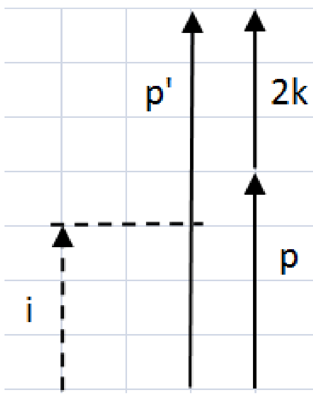
The similar can be proved for P(2n+ δ).

The above again proves that there are an infinitude of twin primes.

Step 7: General twin prime conjecture

The general twin prime conjecture states: “for every natural number k, there are infinitely many prime pairs p and p′ such that p′ – p = 2k. The case k = 1 is the twin prime conjecture.”

Based on the above, we can write: p′ = p + 2k.



We again work out our formula:

$$P(p') = P(p+2k) = \prod_{i=2}^{p'-1}\left[\sin\left(\frac{\pi(p+2k)}{i}\right)\right]$$

$$= \prod_{i=2}^{p'-1}\left[\sin\left(\frac{\pi p}{i}\right).\cos\left(\frac{2k\pi}{i}\right) + \cos\left(\frac{\pi p}{i}\right).\sin\left(\frac{2k\pi}{i}\right)\right] \neq 0?$$

The left term will be 0 because for i=p, the sinus will become 0.
The right term will never become 0 since p/p’ will never become ½ otherwise p’ wouldn’t be prime. Neither will 2k/p or 2k/p’ ever become a natural number for the same reason.

We conclude that the equation will always be different from 0 and hence that we can choose any k to find a p’.

So, an infinite amount of possibilities are available.

Step 8: 2^p–1 is prime if p is prime

We will try to prove this here using our formula:

$$P(2^p-1)=\prod_{i=2}^{2^p-2}\left[\sin\left(\frac{\pi(2^p-1)}{i}\right)\right]$$
$$=\prod_{i=2}^{2^p-2}\left[\sin\left(\frac{\pi 2^p}{i}\right).\cos\left(\frac{\pi}{i}\right)+\cos\left(\frac{\pi 2^p}{i}\right).\sin\left(\frac{\pi}{i}\right)\right]\neq 0?$$

For i=2, the first term will be 0 due to the cosines.
In the second term, the sinus will never be 0 as the argument will never reach kπ.

So, we can write:

$$P(2^p-1)\neq 0\leftarrow\prod_{i=2}^{2^p-2}\left[\cos\left(\frac{\pi 2^p}{i}\right)\right]\neq 0?$$

For all i different from multiples of 2, it is clear that the cosines term will never reach π/2 or 3π/2. So the cosines will never be 0.
For all i that are a multiple of 2, the value in the denominator can be eliminated against the value in the teller. What remains in the teller will be a multiple of 2 as well. So, this cosines value will never be 0 either. Furthermore, i will never reach 2^{p+1}.

So, it is clear that the result will be <>0 and that 2^p-1 will be prime if p is prime.

Step 9: Perfect numbers

Another open question is whether there are infinitely many perfect numbers.

In number theory, a perfect number is a positive integer that is equal to the sum of its proper positive divisors, that is, the sum of its positive divisors excluding the number itself (also known as its aliquot sum). Equivalently, a perfect number is a number that is half the sum of all of its positive divisors (including itself) i.e. $\sigma_1(n) = 2n$.

It is also unknown whether there are any odd perfect numbers.

First of all lets write down the formula for the divisor function:

$$n = \prod_{i=1}^{r} \mathrm{p}_{\mathrm{i}}^{\alpha_{\mathrm{i}}} = \prod_{i=1}^{r} d_i$$

with r the number of distinct prime factors of n, p_i is the ith prime factor, and α_i is the maximum power of p_i by which n is divisible.

Suppose n is a non prime and hence it has r divisors d_i that satisfy the above formula.
Then we can write our formula 1 as follows:

$$P(n) = \prod_{i=2}^{n-1} \sin\left(\frac{\pi n}{i}\right) = 0 => not \text{ Prime}$$

or:

$$P(n) = \sin\left(\frac{\pi n}{2}\right).\sin\left(\frac{\pi n}{3}\right)\ldots.\sin\left(\frac{\pi n}{d_{i=1}}\right)\ldots\sin\left(\frac{\pi n}{d_{i=r}}\right)\ldots\sin\left(\frac{\pi n}{n-1}\right)$$
$$= \prod_{i=1}^{r} \sin\left(\frac{\pi n}{d_i}\right). \prod_{j=2\ \& j\neq d_i}^{n-1} \sin\left(\frac{\pi n}{j}\right)$$

We have separated all the divisors d_i of n in the first term after the equation and all these sinus will be 0. This is the reason why P(n)=0.
The second term is ≠0.

Step 10: even Perfect numbers

In this part, we will only talk about even perfect numbers (2n). We will say something about odd perfect numbers in the next part.

Since in this part, we are talking only about even numbers, it means that 2 is at least a divisor (2n/2=n), together with 1 and itself.

We will show how to calculate the divisors starting from the number of divisors that we want to calculate.

We will use following general presentation as a guide:

n								sum:
i (or di)	1	2	d3	d4	...	dr	n	2n
degrees								
mod(degrees,180)								
times 180 (or ki=degrees/di)	n	dr	dr-1	dr-2		2	1	2n
di x ki	n	n	n	n	n	n	n	

number of divisors: 3

n					sum:
i (or di)	1	2	d3	n	2n
degrees					
mod(degrees,180)					
times 180 (or ki=degrees/di)	n	d3	2	1	
di x ki	n	n	n	n	

$$\begin{cases} 2.d_3 = n \\ 1 + 2 + d_3 = n \end{cases}$$

Solving this gives:

$$\begin{cases} d_3 = 3 \\ n = 6 \end{cases}$$

Which is a known solution.

number of divisors: 4

n						sum:
i (or di)	1	2	d3	d4	n	2n
degrees						
mod(degrees,180)						
times 180 (or ki=degrees/di)	n	d4	d3	d2	1	
di x ki	n	n	n	n	n	

$$\begin{cases} 2.d_4 = n \\ d_3.d_3 = n \\ 1 + 2 + d_3 + d_4 = n \end{cases}$$

This gives:

$$\begin{cases} d_4 = n/2 \\ d_3 = \sqrt{n} \\ \frac{n}{2} - \sqrt{n} - 3 = 0 \end{cases}$$

This gives $n = 1 \pm \sqrt{7}$

So no solutions in natural numbers.

In general, we expect no solutions for even number of divisors (excluding itself!).

<u>number of divisors: 5</u>

n							sum:
i (or di)	1	2	d3	d4	d5	n	2n
degrees							
mod(degrees,180)							
times 180 (or ki=degrees/di)	n	d5	d4	d3	2	1	
di x ki	n	n	n	n	n	n	

$2. d_5 = n\,;\ d_3.d_4 = n\,;\ 1 + 2 + d_3 + d_4 + d_5 = n$

It seems like d_3 can be freely chosen to come to the other unknowns and n should be even.

Solving for n, we get the following formula:

$$n = \frac{3 + d_3}{1 - \frac{1}{2} - \frac{1}{d_3}}\ or\ as\ we\ will\ see\ later: n = \frac{1 + (2 + d_3)}{1 - (\frac{1}{2} + \frac{1}{d_3})}$$

It looks like the only natural solutions for the unknowns are then:

1,2,3,12,18,36 and 1,2,4,7,14,28.

This follows from calculations in Excel:

d3	d4	d5	n	check		
1	-8	-4	-8	-8		
2	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!		
3	12	18	36	36		
4	7	14	28	28		
5	5.333333	13.33333	26.66667	26.66667		formula for B10: =B9+1
6	4.5	13.5	27	27		formula for C10: =E10/B10
7	4	14	28	28		formula for D10: =E10/2
8	3.666667	14.66667	29.33333	29.33333		formula for E10: =(3+B10)/(1-1/2-1/B10)
9	3.428571	15.42857	30.85714	30.85714		formula for F10: =3+B10+C10+D10
10	3.25	16.25	32.5	32.5		
11	3.111111	17.11111	34.22222	34.22222		
12	3	18	36	36		
13	2.909091	18.90909	37.81818	37.81818		
14	2.833333	19.83333	39.66667	39.66667		
15	2.769231	20.76923	41.53846	41.53846		
16	2.714286	21.71429	43.42857	43.42857		
17	2.666667	22.66667	45.33333	45.33333		
18	2.625	23.625	47.25	47.25		
19	2.588235	24.58824	49.17647	49.17647		
20	2.555556	25.55556	51.11111	51.11111		
21	2.526316	26.52632	53.05263	53.05263		
22	2.5	27.5	55	55		

number of divisors: r

we select only r odd so that there is a middle column (after separating 1 and n).

n									sum:
i (or di)	1	2	di	d(r+1)/2	d(r+3)/2	dr-i+2	dr	n	2n
degrees									
mod(degrees,180)									
times 180 (or ki=degrees/di)	n	dr	dr-i+2	d(r+3)/2	d(r+1)/2	di	2	1	
di x ki	n	n	n	n		n	n	n	

$$2. d_r = n\,;\ d_i . d_{r-i+2} = n\ \ for\ i = 3\ to\ \frac{r+1}{2};\ 1 + 2 + \sum_{i=3}^{r} d_i = n$$

We can rewrite n as:

$$n = 1 + \sum_{i=2}^{r} d_i = 1 + \sum_{i=2}^{\frac{r+1}{2}} d_i + \sum_{i=\frac{r+3}{2}}^{r} d_i = 1 + \sum_{i=2}^{\frac{r+1}{2}} d_i + \sum_{i=\frac{r+3}{2}}^{r} \frac{n}{d_{r-i+2}}$$

since:

$$\sum_{i=\frac{r+3}{2}}^{r} \frac{1}{d_{r-i+2}} = \sum_{i=2}^{\frac{r+1}{2}} \frac{1}{d_i}$$

We come to the general formula for perfect numbers:

$$n = \frac{1 + \sum_{i=2}^{\frac{r+1}{2}} d_i}{1 - \sum_{i=2}^{\frac{r+1}{2}} \frac{1}{d_i}}$$

So, we will have to look for solutions of this equation where n becomes a natural number for the chosen divisors.

Remark: other perfect numbers are: 6,28,7056, …

Step 11: Odd Perfect numbers

Another open question is whether there are any odd perfect numbers.

We will first of all present two examples with the perfect numbers 6 and 28:

n	6			$\sigma_1(n)$	12			$\sigma_1(n)$-n	12	aliquot sum	
i (or di)	1	2	3	4	5	6					
degrees	1080	540	360	270	216	180					
mod(degrees,180)	0	0	0	90	36	0					
times 180 (or ki=degrees/di)	6	3	2			1					
di x ki	6	6	6			6					

n	28			$\sigma_1(n)$	56			$\sigma_1(n)$-n	28	aliquot sum			are there any odd perfect numbers?	
i (or di)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
degrees	5040	2520	1680	1260	1008	840	720	630	560	504	458	420	388	360
mod(degrees,180)	0	0	60	0	108	120	0	90	20	144	98.2	60	27.7	0
times 180 (or ki=degrees/di)	28	14		7			4							2

n														
i (or di)	15	16	17	18	19	20	21	22	23	24	25	26	27	28
degrees	336	315	296	280	265	252	240	229	219	210	202	194	187	180
mod(degrees,180)	156	135	116	100	85.3	72	60	49.1	39.1	30	21.6	13.8	6.67	0
times 180 (or ki=degrees/di)														1

We can generalize the above using the following diagram:

n								sum:
i (or di)	1	…	p1	…	p2	…	n	2n
degrees								
mod(degrees,180)								
times 180 (or ki=degrees/di)	n		p2		p1		1	2n
di x ki	n		n		n		n	

Out of the examples we can generalize that:

- the divisors d_i of n will be listed from left to right in an ascending order starting with 1, …, p1, …, p2 until we reach the last (improper) divisor n itself.
- the k_i, which represent the number of times the divisor d_i will go into n, will be listed from left to right in a descending order starting with n, …, p2, …, p1 until we reach 1.
- if there are r divisors then we can say that $d_i = k_{r-i+1}$
- So, the row of $k_i$will be the same as the row of d_i but in opposite order.
-

Since the sum of the proper divisors needs to be n in order to be a perfect number and since we are looking for an n which is odd, it means that except for 1 (which is odd) and n itself, there can only be an even number of odd divisors of n. The easiest way to represent this is by taking 2 odd divisors p1 and p2 in the representation above.

Since n needs to be odd, it can either be a prime (case 1) or a composite (case 2).

Case 1: n is an odd prime, say n=p3

We replace n in the excel everywhere with p3:

case 1

n								sum:
i (or di)	1	…	p1	…	p2	…	p3	2n
degrees								
mod(degrees,180)								
times 180 (or ki=degrees/di)	p3		p2		p1		1	2n
di x ki	p3		p3		p3		p3	

The first row says that 1+p1+p2=p3. This could be possible.
The first column says that 1.p3=p3. This is ok.
Out of the second column, we then conclude that p1.p2=p3.
But since p3 is a prime, this is not possible.
So, this is a contradiction.
Hence case 1 is not possible.

Case 2: n is a composite, say n=k.p2

We replace n in the excel everywhere with k.p2:

case 2: n is an odd composite, say n= k.p2

n								sum:
i (or di)	1	...	p1	...	p2	...	k.p2	2n
degrees								
mod(degrees,180)								
times 180 (or ki=degrees/di)	k.p2		p2		p1		1	2n
di x ki	k.p2		k.p2		k.p2		k.p2	

The first column says 1.k.p2= k.p2. This is ok.
The second column says: p1.p2=k.p2. This is possible. It means k=p1.
The first row says: 1+p1+p2=k.p2. Using k=p1, we then get: 1+k=p2(k-1)
I then follows that p2= (k+1)/(k-1).

We split this up in 2 sub cases: k=2 or k>2.

<u>Sub case a:</u> k=2

This gives: p2=3. It follows then that p1=2 and n=6.
Although this is a perfect number, n is in this case even what is not the solution we are looking for.

<u>Sub case b:</u> k>2 say p1=3

In this case p2 will never reach an integer value any more.

So there are no odd solutions for a perfect number.

<u>An alternative method to prove that there are no odd perfect numbers is as follows:</u>

Once you multiply an odd number with an even number, you will get an even number. This means that an odd number can only have odd divisors.

In order to get a perfect number, the sum of all proper divisors of our odd number must equal our odd number. We can only get an odd number again with odd divisors if we add up an odd number of divisors (1 inclusive). So, 1 exclusive, we have an even number of odd divisors.

<u>Suppose, we have 2 divisors apart from 1:</u>

Then we can write the following 2 equations:

$$n(odd) = 1 + d_1 + d_2$$

$$n(odd) = 1.d_1.d_2 = d_1.d_2$$

In order to get a perfect odd number, we must equal both equations. This will give:

$$1 + d_1 + d_2 = d_1 . d_2$$

Solving this equation for d_2 gives:

$$d_2 = \frac{d_1 + 1}{d_1 - 1}$$

We concluded above that the divisors needed to be odd. So we start with assuming that $d_1$is odd. This however means that $d_1 + 1$ is even and that $d_1 - 1$ is also even.

A first possible solution is for $d_1 = 2$. This results in $d_2 = 3$. However, d_2 odd is not acceptable.

For $d_1 > 2$, $d_2 will\ be < 2$ and for $d_1 \to \infty;\ d_2 \to 1$.

So no real solutions can expected for 3 real divisors.

<u>Suppose, we have r divisors:</u>

Then we can write the following 2 equations:

$$n(odd) = 1 + \sum_{i=2}^{r} d_i = 1 + \sum_{i=2}^{r-1} d_i + d_r$$

$$n(odd) = d_{\frac{(r-1)}{2}} . d_{\frac{(r-1)}{2}} = d_i . d_{r-i+1} = 1 . d_r$$

In order to get a perfect odd number, we must equal both equations. This will give:

$$1 + \sum_{i=2}^{r-1} d_i + d_r = 1 . d_r$$

Solving this equation for d_r gives:

$$1 + \sum_{i=2}^{r-1} d_i = 0$$

So no real solutions can expected for r (odd) divisors.

Step 12: Mersenne numbers

Another open question is: No proof is known whether there are infinitely many Mersenne primes.

A Mersenne prime is any prime that is of the form:

$M_n = 2^n - 1$ where n is a prime itself.

Using our formula:

$$P(n) = \prod_{i=2}^{n-1} \sin\left(\frac{\pi n}{i}\right) <> 0 => \text{Prime}$$

$$P(M_n) = \prod_{i=2}^{2^n-2} \sin\left(\frac{\pi(2^n-1)}{i}\right) = \prod_{i=2}^{2^n-2} \left[\sin\left(\frac{\pi 2^n}{i}\right).\cos\left(\frac{\pi}{i}\right) - \cos\left(\frac{\pi 2^n}{i}\right).\sin\left(\frac{\pi}{i}\right)\right]$$

For i=2, the left term will be 0 and the right term will be <> 0.
So, in general, $P(M_n) \neq 0$ hence, M_n will be prime.

This means that n=3 will be a Mersenne prime.

For i>2 the equation will be prime if:

$$\sin\left(\frac{\pi 2^n}{i}\right).\cos\left(\frac{\pi}{i}\right) <> \cos\left(\frac{\pi 2^n}{i}\right).\sin\left(\frac{\pi}{i}\right)$$

or if:

$$\tan\left(\frac{\pi 2^n}{i}\right) <> \tan\left(\frac{\pi}{i}\right)$$

This will be the case if i is a divisor of n.

Example: M_{12}

$$M_{12} = 2^{12} - 1 = 4095 = 195 * 21$$

The equation:
$\sin\left(\frac{\pi 2^n}{i}\right).\cos\left(\frac{\pi}{i}\right) <> \cos\left(\frac{\pi 2^n}{i}\right).\sin\left(\frac{\pi}{i}\right)$
becomes (for i=21):

(-0.1490).(0.9888)-(0.9888).(-0.1490)=0
Which means that M_{12} is not a prime (because 12 is not a prime).

Generally speaking, M_n will be a Mersenne prime if it has no divisors between 2 and $M_n - 1$.

A theorem about Mersenne numbers states that if $M_p$is prime then the exponent p must also be prime.

Since there are infinitely many prime numbers above n, it means that there will be infinitely many Mersenne primes as well.

Step 13: odd harmonic divisor numbers

Using a small example, we will derive the formula for a harmonic divisor:

Divisors of 28 are: 1,2,4,7,14,28. So, we have 5 proper divisors (=r) and including 28 itself, we will have 6 (=r+1).

Using a small example, we recapitulate the conventions for the variables:

n=28	HM=3=(r+1)/2					∑
i	1	2	3=(r+1)/2	4	5 (=r)	6 (=r+1)	
di	1	2	4	7	14	28	56 (=2n)
ki=n/di	28	14	7	4	2	1	56 (=2n)
ki x di=n	28	28	28	28	28	28	

The harmonic mean (HM) for 28 is:

$$HM(28) = \frac{6}{\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}} = 3$$

So, we can write the general formula for the harmonic mean using our conventions:

$$HM(general\ n) = \frac{r+1}{\sum_{i=1}^{r+1} \frac{1}{d_i}} = \frac{r+1}{\sum_{i=1}^{r+1} \frac{k_i}{n}} = \frac{n.(r+1)}{\sum_{i=1}^{r+1} d_i}$$

This last expression is true because: $\sum_{i=1}^{r+1} k_i = \sum_{i=1}^{r+1} d_i$

n=26	HM=	2.47619			∑
i	1	2	3 (=r)	4 (=r+1)	
di	1	2	13	26	42 (<>2n)
ki=n/di	26	13	2	1	42 (<>2n)
ki x di=n	26	26	26	26	

In the case of perfect numbers, the sum of all the divisors of n equals 2n.
Then the harmonic mean for perfect numbers becomes:

$$HM(perfect\ n) = \frac{(r+1)}{2}$$

Oystein Ore, who defined the harmonic divisor number (also called ore number), conjectured that no odd harmonic divisor numbers exist other than 1. If the conjecture is true, this would imply the nonexistence of odd perfect numbers.

As a matter of fact this is true. If we use our previously derived formula:

$$HM(n) = \frac{n.(r+1)}{\sum_{i=1}^{r+1} d_i}$$

We try to list all the possible scenario's in terms of odd/even:

n	(r+1)	$\sum_{i=1}^{r+1} d_i$	HM	result if integer
even	even	even	even*even/even	even
even	even	odd	even*even/odd	not an integer
even	odd	even	even*odd/even	even
even	odd	odd	even*odd/odd	even
odd	even	even	odd*even/even	even or not an integer
odd	even	odd	odd*even/odd	even
odd	odd	even	odd*odd/even	not an integer
odd	odd	odd	odd*odd/odd	odd

We see that the only chance to have an odd harmonic mean is in the case where all the 3 components of the formula are odd.

We know that an odd number of divisors (r+1) is only possible for squares.

Since every perfect number is harmonic and an harmonic number can only be odd in the case of n being a square but a perfect number cannot be a square it follows that no odd perfect numbers exist.

In the following, we explore possible odd harmonic numbers a little further.

We look at the following example for n=81:

n=81	HM=	3.347107			∑
i	1	2	3	4 (=r)	5 (=r+1)	
di	1	3	9	27	81	121(<>2n)
ki=n/di	81	27	9	3	1	121(<>2n)
ki x di=n	81	81	81	81	81	

Another example for n=9:

n=	9	HM=	2.076923	r+1	∑
	i	1	2	3		
	di	1	3	9	13	<>2n
	ki=n/di	9	3	1	13	<>2n
	ki x di=n	9	9	9		

Lets generalize our form a bit for purpose of finding possible odd solutions. This gives for 2+1 divisors:

n	HM=	n*(r+1)/∑		∑
i	1	2	3 (=r+1)	
di	1	d1	n	1+d1+n
ki=n/di	n	d1	1	1+d1+n
ki x di=n	n	n	n	

So, we get:

$$n = d_1.d_1 = d_1{}^2$$

$$\sum d_i = 1 + d_1 + n = 1 + d_1 + d_1{}^2$$

$$HM = \frac{3.{d_1}^2}{1 + d_1 + {d_1}^2} \rightarrow \left(1 - \frac{3}{HM}\right).{d_1}^2 + d_1 + 1 = 0$$

This gives:

$$d_1 = \frac{-1 \pm \sqrt{1 - 4.(1 - \frac{3}{HM})}}{2.(1 - \frac{3}{HM})} = \frac{-1 \pm \sqrt{-3 + \frac{12}{HM}}}{2.(1 - \frac{3}{HM})}$$

The above equations has only real solutions for HM≤4.
For HM=4, $d_1 = -1$.
For HM=3, $d_1 = not\ determined$
…

We can try the next generalized form for 5 divisors:

n	HM=	n*(r+1)/∑				∑
i	1	2	3	4 (=r)	5 (=r+1)	
di	1	d1	d2	d3	n	1+d1+d2+d3+n
ki=n/di	n	d3	d2	d1	1	1+d1+d2+d3+n
ki x di=n	n	n	n	n	n	

From here, we get:

$$d_3 = \frac{n}{d_1};\ d_2 = \sqrt{n}\,;\ r + 1 = 5\,;\ \sum d_i = 1 + d_1 + d_2 + d_3 + n$$

The harmonic mean then becomes:

$$HM(n) = \frac{n.(r+1)}{\sum_{i=1}^{r+1} d_i} = \frac{5n}{1 + d_1 + \sqrt{n} + \frac{n}{d_1} + n}$$

Since $d_1 < n$, the value for HM will never reach a value of 5. No possible integer solutions lie within this range.

Since solutions for more than 5 divisors can be written using similar formula's as above, we can summarize by saying that no odd harmonic means will ever exist.

Results

It is clear from the above that our formula for differentiating primes from non primes is sufficiently strong to prove the existing open conjectures. This is done using basic trigonometric formula's and some simple interpretations.

Discussions:

It is our strong belief that in the representation of complex numbers an extra factor would be useful to represent the spin. Although in most calculations φ and φ+360 or φ+720 represent the same vector, this might not necessarily always be the case.

The working out of this will however be for a next paper as it seems to be a little complicated due to the rules of group theory involved.

The working out of the relation between divisibility and standing waves could be further developed as well.

Conclusion:

1. Discussing about primes and prime gaps is not so difficult if one uses the right formula to demonstrate prime.
2. The natural numbers are just a special case in the complex plane which seems to be a projection from the complex helix.
3. Divisibility is not about the smaller number fitting into the larger one but about the larger wave from the smaller numbers coming together with the smaller wave from the larger numbers in a standing wave for the different i.

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